STUDENT PERSPECTIVES OF PRODUCTIVE STRUGGLE IN HIGH SCHOOL MATHEMATICS

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Abstract

The purpose of this study was to explore the student experience in productive struggle for thirteen Algebra II and Geometry students, and to explore the connection between productive struggle and advancement in mastery level for nine Geometry students. The results of this study may help provide educators with insight on ways to support students in productive struggle and leverage productive struggle to facilitate student achievement. A qualitative analysis of classroom observations, student interviews, and student work samples revealed a productive struggle progression and an emotional progression as Algebra II and Geometry students work on challenging math tasks. Additionally, the analysis revealed differences in the productive struggle patterns for advancing bubble students and non-advancing bubble students in the beginning and middle phases of the task. Insights from this study support the conjecture that certain aspects of productive struggle are associated with advancement in mastery level, and that explicit teaching of productive struggle types and progressions could serve as a strategy to increase academic achievement.

Keywords: productive struggle, challenging math task, doing math, student growth, student achievement
Dedication

When I began to consider pursuing a doctoral degree, my daughter’s Girl Scout troop needed a troop leader. I was not sure if I should take on both commitments, but I stepped out in faith to do it anyway. I drove to Asheville to take the GRE, and was taken aback when the testing facility was located next to the Asheville Girl Scout Council office. I knew then that I had made the right decision.

This dissertation is dedicated to the girls, volunteers, and parents of Girl Scouts of Southern Appalachians Troop 166. It has been a delight to watch the girls grow in these past three years, and the selfless service of the troop volunteers has been an inspiration. I have enjoyed all our adventures together, and I hope we have many more. I know that you were all meant to be in my life as I pursued this degree. The troop has taught me of the joy of learning and growing, shown me how to be courageous and strong, and reminded me to be a sister to every Girl Scout.
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CHAPTER ONE: Introduction

Background

The face of mathematics education underwent a transformation in the first decade of the 21st century. In 2010, Tennessee and many other states adopted the Common Core State Standards, introducing a focus on deep conceptual understanding alongside procedural fluency in math (Friedberg et al., 2018). Jochim and McGuinn (2016) described how teachers, parents, and students alike faced a significant challenge to accept and transition into this new math mindset. The Tennessee Department of Education abandoned the name Common Core in favor of the more politically neutral label, Tennessee State Standards, but the rigor and conceptual focus of the standards remained (Friedberg et al., 2018). Throughout the transition to the more rigorous, deeply conceptual standards, there remained a strong focus on school accountability through standardized testing, a holdover from the No Child Left Behind and Race to the Top nation-wide school accountability legislation (Jochim & McGuinn, 2016).

Teachers remained mindful of the need for students to achieve top scores on state tests (Camera, 2015; Jochim & McGuinn, 2016), and states and professional organizations offered training and publications to equip teachers to implement the new standards (Booker, 2013; Friedberg et al., 2018). One such publication was the National Council for the Teachers of Mathematics’ (NCTM) 2014 document, Principles to Actions, which outlined eight effective math teaching practices necessary for students to develop deep conceptual understanding and do rigorous math work. One of these eight practices was to support productive struggle in learning mathematics (NCTM, 2014). The researcher aimed to learn more about the student’s experience during productive struggle as the teacher supported the student in challenging math tasks in
Algebra II and Geometry and examined how this might support student achievement on standardized testing.

**Theoretical Framework**

Constructivist learning theory underpinned the theoretical framework for this study. Schiro’s (2013) explanation of constructivist learning theory held that lasting learning happened when learners constructed knowledge through inquiry and personal experience. These experiences involved perceiving stimuli, engaging with the stimuli, and making meaning out of the interaction. Learners’ experiences included reconstructing existing cognitive structure to accommodate new meaning (Schiro, 2013).

Bada (2015) described constructivist learning as driven by the learner’s curiosity and innate predisposition to derive a mental model of the world from his perceptions. Bada (2015) emphasized that the constructivist learning theory centered on the idea that learning was truly *constructed*. Learners constructed new knowledge on the foundation laid by previous learning. Learners used what they already knew to create new understandings. When learners encountered concepts inconsistent with their current understanding, their understanding changed to accommodate the new perspective. Bada (2015) also emphasized the dynamic nature of constructivist learning. To learn constructively, teachers must allow students to actively engage in learning experiences through experiments and real-world problem-solving. Also, constructivist learning included students reflecting on new knowledge and talking about how their understanding was developing (Bada, 2015).

Jones, Jones, and Vernette (2010) described the constructivist math classroom as having a culture where students were active and reflective, using what they knew to develop new ideas and transfer key concepts to new situations. In addition, Jones et al. (2010) described learning as
consistently continuous and always framing new experiences within prior knowledge. Finally, the description included that learning was social, requiring interaction to develop a deep conceptual understanding (Jones et al., 2010).

An essential practice in the classroom used for this study was the leveraging of Vygotsky’s (1978) zone of proximal development (ZPD) to springboard students into new conceptual understandings. Wass and Golding (2014) summarized the use of ZPD as presenting learning opportunities that were slightly too hard for students to do independently, but easy enough to do with careful help. Engaging in learning opportunities in the ZPD brought students to a higher level of independent understanding. Wass and Golding (2014) posited that teaching in the ZPD involves enabling students to do more by developing students’ knowledge, skills, and conceptions. Vygotsky (1978) described this concept as taking students from their actual developmental level to realize their potential level of development through tasks that require some assistance. Wass and Golding (2014) elaborated on the type of assistance warranted to foster student growth within the ZPD. Wass and Golding described this assistance as scaffolding, including support, guidance, advice, direction, or resources. Further, as students become more independent in the task, teachers removed the scaffolds, as the final goal was for students to do the task independently (Wass & Golding, 2014).

The researcher frequently presents new concepts within inquiry learning activities instead of direct instruction in her classroom. With the constructivist framework and ZPD in mind, The researcher examined the student experience of productive struggle as students worked on inquiry-learning activities in the form of challenging math tasks.
Conceptual Framework

Hiebert and Grouws (2007) explained that when a teacher facilitates experiences requiring students to investigate, reason, and do math, the teacher is encouraging productive struggle. Students expended effort to make sense of math and deduce concepts not immediately apparent. This sense-making involved solving problems that were within reach and exploring mathematical concepts that were understandable but not yet solid in the learner’s mind (Hiebert & Grouws, 2007). Warshauer (2015a) determined that as teachers facilitate this, there was a process that occurred, summarized as the Productive Struggle Framework. This framework included four dimensions: Tasks, Student Struggle, Teacher Response, and Outcome (Warshauer, 2015a).

First, in the Tasks dimension, the teacher initiated the process by choosing a task for students with a specific level of cognitive demand. These levels included memorization, procedures without connections, procedure with connections, and doing math. Procedures with connections and doing math were the levels most likely to elicit productive struggle (Warshauer, 2015a). Procedures with connections tasks used procedures to develop deeper levels of understanding and engage with conceptual ideas underlying the procedures (Stein, 2000). Doing mathematics tasks required non-algorithmic thinking and exploration of mathematical concepts. Also, these tasks required metacognition and analysis of solution paths (Stein, 2000).

Next, the Student Struggle dimension denoted the aspect of struggle the student experiences. These included getting started, carrying out a process, giving a mathematical explanation, and expressing a misconception or error. Students tended to experience trouble getting started and carrying out a process in the initial phases of a task, while the struggle to give a mathematical explanation and express a misconception or error tended to materialize at the end
of the task. Students and teachers should not see struggle as an impediment to learning, but rather an opportunity for growth (Warshauer, 2015a).

In the next dimension, there was interaction between student and teacher in the Teacher Response dimension. Herein, the teacher’s response could make or break the cognitive demand of the task. The teacher could interact with the student by directly telling, providing directing or probing guidance, or offering affordance, where the teacher allowed space for students to grapple with the task. Probing guidance and affordance were most likely to maintain a high level of cognitive demand for the task (Warshauer, 2015a).

Finally, there was resolution in the Outcome dimension. This was where students, through their struggle, are either doing math or not. There could be productive struggle, productive low-level struggle, and unproductive struggle in this dimension. If students struggled productively, they developed conceptual understanding throughout the task. Productive low-level struggle occurred when a teacher or peer intervened to remove the struggle, resulting in a correct answer, but lower cognitive demand. Unproductive struggle occurred when the student abandoned the task or was unable to understand the concept at work, or when the teacher changed the nature of the task to be merely procedural (Warshauer, 2015a).

The research questions for this study focused on the student struggle dimension as well as the outcome dimension. One goal was to learn about the patterns of productive struggle that students’ experience—the what, how, and why of their struggle from their perspective. Another goal was to see if and how the struggle leads the students to do math and advance to the next mastery level.
**Statement of the Problem**

The researcher’s school struggled with numeracy scores on state tests beginning in 2014. The Value-Added Growth Measure for 2017-2018 was -6.8 for Algebra II, with a 3-year cumulative average of -3.3 (Tennessee Value-Added Assessment System, 2019). Positive growth measures indicate that students made more than the expected academic progress. Zero growth measures indicate that students made the expected academic progress. Negative growth measures indicate that students made less than the expected academic progress (SAS Institute, Inc., 2019). Therefore, the negative growth scores indicate that students at this school are consistently making less than the expected academic progress. Math department leadership needed to look for strategies to improve student achievement in this area, and engaging students in productive struggle with more frequency was a promising solution (Fung, Tan, & Chen, 2018). The researcher aimed to learn more about the students’ perspective during productive struggle as well as explored how a focus on productive struggle throughout the Algebra II and Geometry courses impacted student mastery of instructional objectives to improve growth and/or achievement scores. Although Algebra II had the lowest scores, Geometry was also a focus, because scores on the state-mandated standardized test, known as TNReady, were available in January for Geometry. The Geometry scores allowed the researcher to examine differences in patterns of productive struggle for students who advanced the next mastery level in comparison to patterns of productive struggle for students who did not advance. In this study, the researcher focused on *bubble* students—those students just on the cusp of scoring at the next achievement level (Blanc et al., 2010).
Purpose of the Study

According to Robelen (2013), Algebra II is hard, probably the most challenging math class many students will take in high school, and many educators considered it a gateway to success in post-secondary education. Students’ willingness to persist in their learning (i.e., engage in productive struggle) impacted their level of achievement in a rigorous math course (Fung et al., 2018). TNReady required students to demonstrate deep conceptual understanding (Tennessee Department of Education, 2019); therefore, it was necessary for students to persevere and engage in productive struggle to develop this conceptual understanding (NCTM, 2014). The researcher desired to know more about the student experience, from the students’ perspectives, as they persisted and productively struggled through tasks to better support learning. In addition, the researcher sought to know more about the connection between productive struggle and advancement in mastery level. This study provided valuable information to the researcher and her colleagues as they implemented challenging math tasks to support productive struggle and boost student achievement.

Research Questions

The research questions guided the researcher to examine the productive struggle experience of students and assess the implication for mastery.

Research Question 1: What patterns of productive struggle occur for bubble students as they work on challenging math tasks supporting mastery of instructional objectives in Algebra II and Geometry in one high school in Tennessee?

Research Question 2: What is the relationship between patterns of productive struggle and mastery level shifts for bubble students in Geometry in one high school in Tennessee?
Rationale for the Study

The state of Tennessee has increased accountability and rigor for teaching and learning since 2010 (Friedberg et al., 2018). Teachers and the administrators who support them searched strategies that engaged students in the current rigorous standards and expectations (Jochin & McGuinn, 2016). NCTM (2014) suggested that engaging students in productive struggle was a powerful method to raise students’ abilities to these standards.

Resources existed to provide teachers with strategies they could use to facilitate productive struggle (Barlow et al., 2018; Betts & Rosenberg, 2016; Cheeseman, Clarke, Roche, & Walker, 2016; Hiebert & Grouws, 2007; Kisa & Stein, 2015; Livy Muir, & Sullivan, 2018; Warshauer, 2015a). Teachers could apply these strategies by rote and see results, but teachers enriched their teaching and students’ learning when they were knowledgeable about their students (Townsend, Slavit, & McDuffie, 2018); therefore, in this study, the researcher took a different focus by examining productive struggle from the students’ perspectives to increase teacher knowledge of students. Through a phenomenographic approach, the researcher aimed to understand how students experienced productive struggle and how it impacted the mastery of instructional objectives. Students had various levels of comfort with inquiry learning activities requiring productive struggle to attain conceptual understanding. In this study, the researcher sought to learn more about the students’ experiences and perspectives as they struggled with mathematical tasks with a high level of cognitive demand. In this study, the researcher focused on *bubble* students because bubble students exist on all but the highest achievement level and have the most significant potential to advance to the next achievement level. There were four levels of achievement, so the researcher could select participants to represent various levels of
students. Then, by examining which students advance to the next level, the researcher could identify characteristics of productive struggle for advancing students.

**The Researcher Positionality Statement**

The researcher has taught mathematics in a rural public high school since 2004, has held an Educational Specialist degree in Brain-Based Teaching since 2011, has served as department chair since 2016, and has served on the school’s teacher leadership team since 2018. The researcher had the opportunity to participate in trainings as Tennessee transitioned to Common Core State Standards, and later to the Tennessee State Standards, beginning in 2012 and continuing through 2019. Challenging math tasks, rigor, *doing math*, and productive struggle have been common themes across these trainings, and the researcher has made a concerted effort to implement these practices in the classroom with some success and much enjoyment. The topic and information gathered in this study informed the researcher’s practice as well as that of other math teachers to refine best practices with a student-centered perspective. The role of the researcher was to ensure that the study, data collection, observation, and interview process were free from bias. The researcher gathered and organized data to reveal the students’ perspective throughout challenging math tasks requiring productive struggle, using the research questions as a guide.

**Limitations and Delimitations**

Limitations for the study included grade level and timing of classes during the school day. Because the selected students were from the researcher’s class roster, only students in grades 9, 10, and 11 participated (because the researcher has no grade 12 students). Further, the researcher’s class schedule determined that periods 2, 4, and 5 would participate in the study during the 5-period school day. Period 1 is the researcher’s planning period, and period 3 is a
student elective course. This class schedule meant that the researcher observed many of the students selected late in the school day, during the final two class periods. Students may have been fatigued at this point in the school day and may have been less likely to respond well to a challenging math task.

Delimitations for this study included the criteria for selection of pre-determined participants from the researcher’s class roster. The researcher selected only bubble students for the study. These students were defined as those scoring 7 or fewer raw score points below the minimum score for the next achievement level on the prior year’s TNReady end-of-course exam. For Algebra II, level 2 students scored at least 304, level 3 students scored at least 321, and level 4 students scored at least 342 on the prior year’s exam. Bubble students were those that scored between 296 – 303, 313 – 320, and 334 – 341, respectively. The researcher identified eight Algebra II students using this metric. Four of these eight students participated in the study. For Geometry, level 2 began at 306, level 3 at 326, and level 4 at 345; therefore, bubble students were those who scored 300 – 305, 319 – 325, and 337 – 344, respectively. The researcher identified 11 Geometry students as bubble students using this metric; nine of these participated.

**Definitions of Terms**

**Bubble student.** Blanc et al. (2010) defined bubble students as students who have the highest likelihood of moving to the next level of performance (from below basic to basic or from basic to proficient) thereby increasing the probability that the school will meet its state accountability goal, known as Adequate Yearly Progress (AYP) (Blanc et al., 2010).

**Doing math.** Van de Walle, Karp, and Bay-Williams (2016) defined doing mathematics as students moving beyond the memorization of math facts and algorithms and instead engaging in the types of thinking that mathematicians do.
Important mathematics. Hiebert and Grouws (2007) defined important mathematics as mathematical skills and concepts related to the understandings and connections between concepts which underpin deep mathematical learning, beyond skills of simple computation.

Productive struggle. The NCTM (2014) defined productive struggle as students delving “more deeply into understanding the mathematical structure of problems and relationships among mathematical ideas, instead of simply seeking correct solutions” (p. 48). Confusion and mistakes occur along the path to learning and deep understanding (NCTM, 2014).

Zone of Proximal Development. Vygotsky (1978) defined ZPD as an educational target just beyond what a student could independently accomplish when provided appropriate support.

Organization of the Document

This document consisted of five chapters. Chapter 1 provided background information and outlined the purpose and significance of the study as well as the theoretical and conceptual frameworks. Also, the chapter stated research questions, along with limitations, delimitations, and definitions. Chapter 2 included a literature review, summarizing previous research on the topic of productive struggle. Chapter 3 explained the study methodology. Finally, Chapters 4 and 5 discussed the results, conclusions, and implications of the study.
CHAPTER TWO: Literature Review

In this chapter, the researcher summarized the literature on productive struggle, integrating key findings and conclusions from studies on student struggle, challenging math tasks, and teacher techniques to support student struggle. The literature extensively discussed benefits of productive struggle as well as teaching materials and techniques which support productive struggle. Further, the chapter presented components of productive struggle, such as sense-making and doing math. In addition, the chapter identified types of student struggle from a researcher’s perspective. One of the goals of this dissertation was to explore the types of student struggle from the student’s perspective, using previously identified struggle types as an initial framework, and using research-based teaching strategies and materials to cultivate a classroom where productive struggle is the norm.

Introduction to Productive Struggle

Researchers have determined that effective math teaching, which developed students’ conceptual development as well as mathematical proficiency, included attention to connections among ideas, facts, and procedures, as well as engagement of students in productive struggle (Hiebert & Grouws, 2007; Kapur, 2016; Warshauer, 2015a). Specifically, productive struggle occurred when students expended effort “to make sense of mathematics, to figure something out that is not immediately apparent” (Hiebert & Grouws, 2007, p. 387), particularly in the context of solving problems that required students to grapple with key ideas that were understandable, but only beginning to take shape in the learner’s mind (Hiebert & Grouws, 2007). The National Council for Teachers of Mathematics (NCTM) (2014) stated that productive struggle involved students delving “more deeply into understanding the mathematical structure of problems and relationships among mathematical ideas, instead of simply seeking correct solutions” (p. 48).
NCTM (2014) researchers stated that the process begins by presenting students with a task that was attainable, yet unsolvable by a rote procedure or previous knowledge. The students’ solution path was unknown at the beginning, and confusion and mistakes arose along the path to learning and deep understanding (NCTM, 2014). Solutions were within reach, yet required students to investigate beyond memorization or repetition of previously taught procedures (Hiebert & Grouws, 2007).

As students worked, teachers supported them without hijacking their thinking (NCTM, 2014). The NCTM (2014) stated that teachers must determine what students understood, determine what was confusing for students, and then carefully craft questions to help students progress without telling them outright what to do. When teachers utilized such questions, students then refined, combined, and modified knowledge to obtain a solution or reach a goal and developed a strong conceptual understanding in the process. The process was not easy—it was a struggle—but there was a sense of accomplishment in the end—it was productive (NCTM, 2014). Other researchers observed that challenge and resulting success led to increased levels of dopamine in the brain, a neurotransmitter accompanied by a sense of pleasure and relaxation (Kienast et al., as cited in Willis, 2010). These pleasurable feelings resulted in the intrinsic motivation needed to pursue an achievable challenge when engaging in a task for productive struggle (Willis, 2010).

Researchers considered a struggle productive if lasting learning and connections between concepts were achieved (Kapur 2016, Warshauer 2015a). The struggle can result in a correct, concrete answer to a math problem (productive success) or not (productive failure); success is not a requirement in this sense for a struggle to be productive as described by Kapur (2016), who based descriptions on the intended instructional design. Warshauer (2015a) described such
struggles as being productive or productive at a lower-level based on how student-teacher interactions resolved the struggle.

According to Kapur (2016), productive success and productive failure were the ideal instructional design conditions because they were productive in developing long term conceptual understanding. Kapur (2016) stated that productive success maximized short term performance and long term learning, often involving problem-solving and guided inquiry learning activities with answers attainable for students as well as enough exploratory value that students grappled with key mathematical ideas. There was one right answer that students found by exploring the problem with reasoning and sense-making as they deepened their mathematical understanding (Kapur, 2016). Kapur (2016) used productive failure to describe conditions that maximized long-term learning without maximizing short-term performance. These conditions engaged students in solving problems involving unlearned concepts, and then followed up with instruction on the concept. There was no one right answer, but students learned from the process of trying to attain a solution (Kapur, 2016). In this design, students used their prior knowledge to generate solutions, which may or may not be correct, but the process of generating these solutions primes their minds to learn the concept through instruction. As in productive struggle, failure and confusion occurred in productive failure (Kapur, 2016). Both productive success and productive failure were productive in that students achieved the goal of lasting learning and deep conceptual understanding (Kapur, 2016). Productive struggle encompasses Kapur’s (2016) concepts of productive success and productive failure.

Warshauer (2015a) described productive struggle as occurring when students persist through a task with a high level of cognitive demand. She identified three categories of struggle: productive, productive at a lower-level, and unproductive, focusing on the resolution of the
struggle to determine its categorization. According to Warshauer (2015a), the interaction between teacher and student played a role in maintaining or reducing the level of cognitive demand of the task as the student worked to resolution. In the ideal case—a productive resolution—the teacher maintained the high cognitive level of the task, and the student persisted through the task while maintaining responsibility for the thinking (Warshauer, 2015a), similar to Kapur’s (2016) productive success and productive failure. The teacher expanded on the student’s reasoning without taking control, insisting that students justify their response. Through attempts to justify their response, students resolve their misconceptions and come to a deeper level of conceptual understanding (Warshauer, 2015a).

In a productive at lower-level struggle resolution, the student reached a conclusion with a teacher or peer intervention, which lowered the level of cognitive demand of the task because the student’s thinking became patterned after that of a teacher or peer (Warshauer, 2015a). The teacher or peer may have suggested their preferred solution path, and the student followed it. Though they came to a correct solution, the intervention removed the student’s opportunity to develop meaningful conceptual understandings, as they did not have to think as deeply to find a solution (Warshauer, 2015a).

Another way to understand the concept of productive struggle was to consider how it was situated within the context of unproductive success and unproductive failure (Kapur, 2016). Warshauer (2015a) determined that the teacher’s questions or instructions could lower the level of cognitive demand of the task by over-directing the students’ thinking, resulting in a correct answer without much cognitive effort. Too much direction from the teacher could remove the opportunity for productive struggle could (Warshauer, 2015a). Kapur (2016) described two unproductive learning design conditions (i.e., short-term and long-term) that had a low level of
cognitive demand for students. Unproductive success maximized success in the short-term but failed to establish long-term learning. This type of instruction involved memorization of procedures and drill-and-practice without connections to conceptual understandings. For example, students might have used an algorithm correctly to solve a multi-digit multiplication problem, while having little concept of the reasoning behind the algorithm or the place values of the numbers in each step (Kapur, 2016). Perhaps all the students answered the classwork correctly by following the algorithm, but none can explain the conceptual underpinnings of the algorithm, nor can they replicate the results the next day without reviewing how to do the problems. Their one-day success was unproductive in developing true understanding (Kapur, 2016). While Kapur (2016) focused on instructional design, Warshauer (2015a) focused on struggle resolution. Warshauer (2015a) described an unproductive struggle resolution as resulting when the student abandoned the task or the task was reduced by teacher or peer intervention to a procedural exercise, similar to Kapur’s (2016) unproductive failure or unproductive success (Warshauer, 2015a). The teacher or peers may have suggested a specific algorithm, and the student utilized it without considering why it works or how (Warshauer, 2015a).

Finally, unproductive failure maximized neither short-term performance nor long-term conceptual understanding (Kapur, 2016). Perhaps the task was too difficult or the teacher provided insufficient scaffolding (Kapur, 2016). Alternatively, the student may have given up (Warshauer, 2015a), perhaps because the task was beyond the student’s realm of achievable challenge (Willis, 2010). Kapur (2016) posited that pure discovery learning, with no teacher guidance, was an example of unproductive failure.
Even these descriptions of the antithesis of productive struggle provided information about productive struggle by revealing what it was not. First, it was not as struggle for the sake of struggle alone, as in Kapur’s (2016) unproductive failure or Warshauer’s (2015a) unproductive resolution. Also, it was not merely answer-getting, like Kapur’s (2016) unproductive success and Warshauer’s (2015a) productive at a lower-level resolution. Instead, productive struggle was purposeful and relevant to classroom learning goals and has offered students the opportunity to build conceptual understanding that furthered their long-term learning (Kapur, 2016; Warshauer, 2015a).

Productive struggle was an incredibly worthwhile goal, but research has revealed it to be intensely challenging for both teachers and students. Teachers, often natural-born leaders, had to shift the responsibility for thinking and directing from themselves and to the student (Betts & Rosenberg, 2016). Students had to think more deeply than before (Betts & Rosenberg, 2016). An essential Math Practice Standard in the Common Core Standards included that students were to “make sense of problems and persevere in solving them” (National Governors Association [NGA] Center for Best Practices, 2010). Students were to do the sense-making and persevering, while teachers facilitated student efficacy in these skills as a foundation for productive struggle (NGA, 2010). This approach was quite different from traditional instructional methods in which students received information from teachers and then later repeated what their teachers directly taught. Productive struggle has demanded deeper cognitive effort from students, as well as perseverance (NGA, 2010). Researchers identified developing students’ ability to persevere in a task as a necessary standard of practice because, as in real-world situations students eventually faced, the first or second strategy attempted may not yield a solution in a task rigorous enough to be considered effective for promoting learning (NGA, 2010). Perseverance, therefore, was a
required precondition for student engagement in productive struggle (Betts & Rosenberg, 2016), and was necessarily interwoven with the Common Core standards (NGA, 2010).

**Student Struggle Types**

Students exhibited struggle requiring perseverance on various levels when engaging in any new task. Warshauer (2015b) observed that students may appear confused, reach an impasse, or struggle to make sense of an answer. The challenge for the teacher was to think of struggles “not as impediments to learning but as opportunities for deepening students’ understanding of mathematics” (Warshauer, 2015b, p. 393). When teachers recognized student struggle types, the teachers could support students in persisting through the struggle (Warshauer, 2015b).

Warshauer’s (2015a) study identified specific types of student struggles visible to an observer as students worked on challenging math tasks. The general procedure for implementing tasks was to have students first attempt the task independently, then work with a partner or small group, and finally take time for class discussion and presenting student work (Warshauer, 2015a). Warshauer (2015a) developed the productive struggle framework while studying student struggle in a middle school math classroom as students completed tasks related to ratios and proportions, but this framework has been applied and proven useful in later studies of varying contexts. For example, Zeybek (2016) studied the struggle types of pre-service middle grades teachers (PSTs) on a high-level task through an embedded exploratory case study involving videoing classroom instruction and conducting group interviews. During classroom instruction, PSTs completed a task requiring them to divide a rectangular cake and its icing evenly between three party guests (Zeybek, 2016).
One type of struggle observed by Warshauer (2015a) was a struggle to get started. In this type of struggle, students voiced confusion about what the task asked them to do, claimed they had never seen or had forgotten how to do this type of problem, or simply did no work at all. In Warshauer’s (2015a) study, this type of struggle occurred 24% of the time. PSTs in Zeybek’s (2016) study also experienced struggle to get started, voicing confusing and becoming hesitant to write anything on their paper, eventually asking questions of the instructor to attempt to resolve their struggle.

The second type of struggle was the struggle to carry out a process (Warshauer, 2015a). Process struggle was the most frequent type of struggle Warshauer observed, occurring 33% of the time. Students may have formulated a plan, but reached an impasse, such as when the numbers became more difficult in a problem (Warshauer, 2015a). Other impasses occurred when students struggled to carry out an algorithm, made numerical mistakes, could not recall a formula, or could not connect a concept to a procedure. These struggles occurred in the initial phases of the task, either when students began working individually or as students began work with a group (Warshauer, 2015a). Zeybek (2016) observed this process struggle when PSTs had difficulty connecting their procedural knowledge of calculating area and perimeter to the conceptual nature of the non-routine task, showing that having procedural knowledge was no guarantee of being able to accomplish a high cognitive demand task.

Two more types of student struggle occurred in the latter part of the tasks in Warshauer’s (2015a) study, as students continued working with their group or attempted to present their results. The third type of student struggle observed by Warshauer (2015a) was a struggle to explain or make sense of their work. Students expressed confusion about how or why their work led to the answer as they share with small groups or the class. They knew they had the right
answer but were unable to identify why the process worked (Warshauer, 2015a). Similarly, in Zeybek’s (2016) study, some members of the group wanted additional justification for solutions that other members devised. Warshauer (2015a) observed that some also struggled to verbalize thinking and justify strategies. The students knew what they were thinking but struggled to explain it when the teacher attempted to elicit a verbal explanation. This type of struggle surfaced because teachers expected students to explain their work—without this expectation, this struggle and opportunity for growth would not have been present (Warshauer, 2015a). Thus, the expectation that students explain and justify their solutions was paramount to facilitating productive struggle (Warshauer, 2015a; Zeybek, 2016). Finally, Warshauer (2015a) identified a struggle to express misconceptions and errors. The misconception and error struggle occurred when students used mathematical misconceptions as a basis for problem-solving, or carried an error through the solution path. In Warshauer’s (2015) study, struggle with misconceptions of content knowledge and number sense centered on probability, fractions, and proportions because of the topic of the tasks. PSTs expressed misconceptions in Zeybek’s (2016) study related to identifying inclusive relationships among quadrilaterals, particularly struggling to accept that a square is a rectangle. This misconception became a hindrance for proceeding with the task (Zeybek, 2016). These misconceptions surfaced when teachers pressed students to justify their solutions and strategies, again illustrating the importance of justification in facilitating productive struggle (Warshauer, 2015a).

Researchers have found it to be the responsibility of the teacher to generate opportunities for some types of student struggle (Warshauer, 2015a; Zeybek, 2016). When students had to justify and explain their work, deep thinking occurred, and they corrected their misconceptions. Furthermore, explanations became just as valued as correct answers, which cultivated a
classroom environment and culture conducive to productive struggle (Warshauer, 2015a; Zeybek, 2016).

### Doing Mathematics

An essential purpose of engaging students in productive struggle has been to have them actively doing mathematics. Van de Walle, Karp, and Bay-Williams (2016) explained that the idea of doing mathematics was to take students beyond the memorization of math facts and algorithms and to have them actively engaged in the types of thinking that mathematicians do. These researchers explained doing math using active verbs such as explore, explain, construct, develop, and predict, and required students to engage in the science of patterns and order using logical reasoning and sense-making (Van de Walle et al., 2016). Researchers do not consider memorizing multiplication tables to be doing math. Instead, students should actively use models and various explorations to come to an understanding of what multiplication is (Van de Walle et al., 2016).

Researchers determined that teachers could present tasks to students that all them to do math. Stein (2000) described these doing math tasks. These tasks did not suggest specific algorithms or procedures. Instead, students could use multiple solution methods to solve the problems. The tasks required students to explore the problem situation and reflect on multiple strategies for a solution. For example, a task may have required students to compare two scenarios involving mathematical computations and make a judgment about the situations, justifying their choice with mathematics (Stein, 2000). To do this, students must have made sense of the problem scenarios and developed a justifiable solution strategy without the teacher giving them a prescribed plan. These experiences pushed students to actively explore, explain, construct, develop, or predict to construct a response (Stein, 2000). When students engaged in
these thought processes, they acted as mathematicians—doing math (Stein, 2000; Van de Walle et al., 2016).

**Sense-making**

The NGA included sense-making as a component of the first Math Practice Standard, “Make sense of problems and persevere in solving them” (NGA, 2010, p. 6). NCTM (n.d.) described sense-making as developing understandings of concepts by connecting them with prior knowledge and suggested that sense-making was an essential support for procedural fluency, as it ascribes meaning to otherwise meaningless algorithms. For example, a thorough exploration of the distance formula through a rich math task helped students connect the formula to the Pythagorean Theorem. The connection to prior knowledge increased the likelihood that students applied the formula correctly to solve problems (NCTM, n.d.). Teachers who employed sense-making allowed students to explore concepts through personal reasoning to develop rules and algorithms instead of presenting the rules and algorithms to students when they begin to teach a topic (Choppin, Clancy, & Koch, 2012). Researchers called this process progressive formalization, and it is an integral part of sense-making (National Research Council [NRC], 2001, as cited in Choppin et al., 2012).

Mueller, Yankelewitz, and Maher (2011) examined sense-making as it relates to positive dispositions toward mathematics and motivation. The researchers determined that understanding resulted from the sense-making in which the students persisted as a result of their intrinsic motivation and positive disposition towards math. The researchers videoed students engaged in challenging math tasks and analyzed the reasoning that occurred as students made sense of the concepts. Students were encouraged to build models, and these models became the basis for
sense-making. In addition, students were encouraged to share solutions and justifications with small groups and the class (Muller et al., 2011).

During these discussions, Muller et al. (2011) observed that when reasoning contradicted a rule that students recalled, students did not merely accept the rule and move on. Instead, students grappled with the discrepancy and remained engaged with the task, using their models to make sense of the problem. Students relied on reasoning rather than memorized facts, demonstrating flexibility in thinking in the journey toward sense-making (Muller et al., 2011). Choppin et al. (2012) also focused research on tasks that allowed students to use informal reasoning before developing algorithms. In both studies, by maintaining confidence in reasoning, students demonstrated self-efficacy, which gave them self-confidence and autonomy to develop an understanding that adjusted previously memorized facts (Choppin et al., 2012; Muller et al., 2011).

Autonomy motivated students to continue their work until they were convinced their solution and strategy made sense (Muller et al., 2011). In summary, Mueller et al. (2011) found that challenging math tasks, supported by an optimal environment, student collaboration, mathematical tools, and teacher variables led students to develop positive dispositions towards math characterized by intrinsic motivation, self-efficacy, and autonomy. These positive dispositions allowed students to persist in reasoning and sense-making on the challenging tasks, resulting in deep conceptual understanding (Mueller et al., 2011).

Researchers explored the significance of independent work coupled with peer interactions to generate sense-making. Moss and Lamberg (2016) presented a framework for three levels of sense-making in mathematics classrooms used to facilitate meaningful discussion. The framework included Making Thinking Explicit, Exploring Each Other’s Solutions, and
Developing New Mathematical Insights (Lamberg, as cited in Moss & Lamberg, 2016).

Students began by working independently on a challenging math task and generating potential solutions before working small groups. Next, in the Making Thinking Explicit phase, students shared their group thinking with the entire class, establishing a foundation for further discussion and exploration (Moss & Lamberg, 2016).

After that, in the Explore Each Other’s Solutions phase, the teacher asked the class to compare groups’ solution paths. The teacher did not validate or reject any answers at this point, but instead left room for learners to develop their own conclusions regarding the reasoning. The teacher called on some groups to defend their solution, the focus being the reasoning and defense, not the correct answers (Moss & Lamberg, 2016).

Then, in the Developing New Mathematical Insights, teachers asked students to discuss all the groups’ answers with their group. This discussion centered on making comparisons between solutions and identifying misconceptions. The teachers used this time to circulate around the room to assess student understandings and identify the big ideas students discussed (Moss & Lamberg, 2016). Choppin et al. (2012) similarly observed students discussing peer’s explanations in order to make sense of the mathematics and noted that the teacher’s role was to orchestrate the discussions. Finally, the whole class discussed and summarized the big ideas generated by the discussion of the groups’ solutions (Lamberg, as cited in Moss & Lamberg, 2016). Each level of this framework connected to the previous level, allowing students to make connections to prior knowledge and learning, as described in NCTM’s (n.d.) definition of sense-making.
Challenging Math Tasks

For students to experience productive struggle, the teacher must present them with challenging math tasks and respond to their struggles in ways that maintain the level of cognitive demand of the task (Kapur, 2016; NCTM, 2014; Stein, 2000; Warshauer, 2015a). Willis (2010) explained that students remain engaged in learning when tasks present an achievable challenge. An achievable-challenge task required students to exert mental effort, but not to the point of frustration. If the task was so frustrating that students wanted to give up, they were not appropriately engaged in the task, or if the task was too easy, the students did not struggle with it and failed to achieve the deep learning that was possible through productive struggle (Willis, 2010). Hiebert and Grouws (2007) posited that Vygotsky’s (1978) ZPD provides a starting point for choosing appropriate tasks. Struggle was likely to be productive for tasks situated within the ZPD. These tasks involved just enough challenge for solutions to be attainable and resulted in students deducing or uncovering novel information. The novel concepts were only accessible if the implementation of the task maintained the level of cognitive demand (Hiebert & Grouws, 2007).

Stein (2000) presented a task analysis guide, now widely used by educators to determine the cognitive demand of a math task. The guide divided tasks first into lower-level and higher-level demands, then divided each of these levels again into subgroups, to create a four-part hierarchy of tasks. According to Stein’s (2000) task analysis guide, lower-level demand tasks included Memorization and Procedures without Connections. Memorization tasks, the lowest level of cognitive demand, involved memorizing or reproducing previously learned facts, rules, or formulas. A connection to concepts underlying the items reproduced was not established. The next step up was Procedures without Connections tasks. These involved the use of
previously learned algorithms to produce correct answers. No explanation was required, and no connection between the algorithm and conceptual understanding was established (Stein, 2000). These two lower-level types of tasks were not where true productive struggle occurred. Students may still have struggled to utilize algorithms correctly and access memorized math facts to produce answers, but because the struggle did not elicit a deeper understanding of the connection between ideas, procedures, and meaningful mathematical concepts, the struggle was not productive struggle (Kapur, 2016; Stein, 2000; Warshauer, 2015a). Lower-level demand tasks fit with Kapur’s (2016) description of unproductive success, in which students obtained correct answers relatively quickly without any deep conceptual understanding developed.

Stein (2000), however, described higher-level demand tasks as those that do afford students the opportunity for productive struggle, as the hallmark of these tasks was developing deep mathematical conceptual understanding. Stein’s (2000) guide presented two types of high-level demand tasks: Procedures with Connections and Doing Math. Procedures with Connections tasks required students to use procedures to develop conceptual understanding. Teachers or peers may have suggested or implied the pathways, and the teacher may have used multiple representations or shown within the task itself (Stein, 2000). Mindlessly following procedures was not enough for students to complete the task; students must have engaged with the concepts underlying the procedures, expending some degree of cognitive effort (Stein, 2000). Stein’s (2000) Procedures with Connections paralleled Kapur’s (2016) productive success learning design. The highest level of demand for tasks was the Doing Mathematics tasks. According to Stein (2000), tasks at the Doing Math level had no set procedure, but instead required the student to engage relevant knowledge and monitor cognitive processes as they analyze the task, which had no predictable solution path. Doing Math tasks involved
considerable cognitive effort and may have caused anxiety and frustration for the student (Stein, 2000). This type of struggle had the characteristics of true productive struggle (Kapur, 2016; Stein, 2000; Warshauer, 2015a). The solution path was unknown; failure and the opportunity to learn from the task were likely to occur, aligning with Kapur’s (2016) description of productive failure.

Stein’s (2000) task analysis framework offered a useful rubric for assessing existing tasks to determine their expected level of cognitive demand. Likewise, Sullivan (as cited in Livy et al., 2018) suggested five requirements that a task must meet to be considered a challenging mathematical task within Stein’s high-level categories. These five requirements provided more clarification for teachers focusing on high-level tasks, or revamping low-level tasks into high-level tasks. First, the task required students to plan their approach by sequencing multiple steps. Students should have engaged in metacognition as they formulated and proceeded with their plan (Sullivan, as cited in Livy et al., 2018). Second, the task required students to make connections among math concepts as they processed multiple pieces of information. Students should have accessed and built on prior knowledge. Third, the task required students to choose their strategies and goals. The implementation of the task does not offer a solution path; students must have relied on independent reasoning to develop a strategy and decide on an acceptable solution for the problems in the task (Sullivan, as cited in Livy et al., 2018). Fourth, the task required students to spend a significant amount of time on the task and make their thinking visible by writing it down. Teachers, therefore, must have allowed plenty of time for students to grapple with the task (Sullivan, as cited in Livy et al., 2018). Fifth, a challenging math task required students to explain strategies and justify thinking to the teacher or peers. The value of
the task was not the arrival at the final answer, but rather, the justifiable process to arrive at the answer (Sullivan, at cited in Livy et al., 2018)

For teachers designing new high-level math tasks to promote productive struggle, Cognitive Load Theory (CLT) has been an informative perspective through which to view task design. Sweller (2010) explained that the number of interacting elements the brain needs to process simultaneously makes up the cognitive load. Russo and Hopkins (2017) developed a framework for designing challenging math tasks using cognitive load theory. Russo and Hopkins’s (2017) framework focused on the intrinsic cognitive load, germane cognitive load, and extraneous cognitive load. Sweller (2010) described these aspects of cognitive load. Sweller (2010) describes intrinsic cognitive load as the extent of interactions of elements included in a learning task as well as the learner’s skill with similar tasks. Extraneous cognitive load, according to Sweller (2010), was also based on the interactions of elements in the task, but it has been described as the cognitive load wasted due to poor instructional design. The difference between intrinsic and extraneous cognitive load in the context of a challenging math task was thought of in terms of the lesson objective (Sweller, 2010). An element of the task which connected to the lesson objective was part of the intrinsic cognitive load, while an element of the task unrelated to the lesson objective was within the extrinsic cognitive load. Finally, the germane cognitive load was the learner’s working memory resources used to tackle the intrinsic cognitive load necessary for the task (Sweller, 2010).

Russo and Hopkins’s (2017) process was called the Cognitive Load Approach to Shaping and Structuring Challenging Tasks (CLASS Challenging Tasks). Cognitive load theory implied that teachers should strive to minimize extraneous cognitive load, maximize the germane cognitive load, and optimize intrinsic cognitive load when developing challenging math tasks.
The CLASS Challenging Tasks framework provided a tool for educators to use to apply cognitive load theory concepts to the development of challenging math tasks to elicit productive struggle.

Russo and Hopkins’s (2017) CLASS Challenging Tasks framework began with identifying the primary learning objective to maximize germane cognitive load. The second step was to develop a problem-solving task that met the primary learning objective. Russo and Hopkins (2017) described appropriate tasks were described as engaging, having multiple solution pathways, involving multiple steps, and requiring significant time to solve—in lockstep with Stein’s (2000) Doing Math tasks as well as Sullivan’s (as cited in Livy et al., 2018) five requirements of a challenging math task. The third step was to ascertain potential secondary learning objectives embedded in the task—knowledge and skills students will be expected to possess or develop by working on the task. After these secondary objectives were identified, step four was to sort them into relevant intrinsic cognitive load items and less relevant extraneous cognitive load items (Russo & Hopkins, 2017). Cognitive load theory implied that it was not efficient to expect students to simultaneously develop unrelated capabilities (Russo & Hopkins, 2017; Sweller, 2010). The less relevant learning objectives may be outside the scope of the lesson and would only serve to create unnecessary extraneous cognitive load; therefore, step five was to redesign the task to remove the potential for extraneous cognitive load, taking the focus off the less relevant objectives. Teachers could redesign tasks many ways (e.g., giving students additional information or providing students with a table to assist in organizing and representing the problem) (Russo & Hopkins, 2017). Step six was to develop enabling and extending prompts to optimize the intrinsic cognitive load for various levels of students. The purpose of enabling and extending prompts was to maintain an appropriate level of cognitive
demand by managing the interacting elements within the cognitive load. The prompts may have altered the task for individual students, but they still had the opportunity to meet the objective because the task became accessible, and they could participate in the discussion component of the lesson as well as explore key mathematical concepts (Russo & Hopkins, 2017). The final step of the process was to include a lesson summary reinforcing the primary learning objective to maximize the germane cognitive load. Russo and Hopkins (2017) suggested restating the primary learning objective and showing a sample of student work which demonstrates mastery of the objective. In Russo and Hopkins’s (2017) framework, the previous six steps were planned before the lesson begins; the student work sample was developed during the lesson.

The expectation for challenging math tasks has been that all students will persist toward a solution. Math tasks were not simply enrichment tasks for high-level learners; tasks allowed all learners to think and learn at a high level (Cheeseman et al., 2016; Livy et al., 2018). For this reason, teachers should have designed tasks to be accessible to all students (Livy et al., 2018). At times, it has been necessary and beneficial to provide scaffolding within the written tasks to give all students an entry point, especially those with diverse learning needs. Townsend et al. (2018) suggested bold fonts and parentheses for clarification within prompts as well as providing blank tables and graphs so students can focus on analysis rather than muddling through the creation of an appropriately scaled chart. Also, structures such as \( y = \)____ and (_____, _____) could clarify response expectations. Such in-text scaffolding increased students’ motivation to continue grappling with a challenging task and overcome struggles (Townsend et al., 2018). In-text scaffolding related to step five of Russo and Hopkins’ (2017) CLASS Challenging Task’s framework, which took steps to reduce the extraneous cognitive load of the task. Livy et al. (2018) suggested pre-planning enabling prompts for students experiencing difficulty as well as
extending prompts for students who complete a task quickly, always allowing students to attempt the original task before giving an enabling prompt, as in step six of Russo and Hopkins’ (2017) CLASS Challenging Task’s framework. Teachers provided these prompts on paper to students on an as-needed basis and gave teachers another tool to assist students besides direct teaching (Sullivan et al., 2015).

**Teaching Strategies to Maintain Cognitive Demand**

After the teacher presented a task with high cognitive demand to students, the teacher must maintain the level of cognitive demand for the student to experience true productive struggle (Kapur, 2016; NCTM, 2014; Stein, 2000; Warshauer; 2015a). Kisa and Stein (2015) indicated that despite the relative ease with which teachers can set up high-level tasks, it was difficult to ensure the teacher maintained the intended level of cognitive demand throughout the time expended on the task. Teachers faced the dual challenge of attending to student thinking while monitoring their own actions to avoid reducing the level of cognitive demand (Kisa & Stein, 2015).

Hiebert and Grouws (2007) summarized features of teaching that promoted conceptual understanding, maintaining the level of cognitive demand of tasks: teaching attended to explicitly to mathematical concepts and the connections between them and teaching that engaged students in struggle with important mathematics (Hiebert & Grouws, 2007).

**Attending to mathematical concepts.** The first feature, attending to concepts, involved “treating mathematical connection in an explicit and public way” (Hiebert & Grouws, 2007, p. 383). Practically speaking, teachers engaged students in discussing the mathematical meaning of procedures, comparing solution strategies, and framing the purpose of a lesson within the broader sequence of learning (Hiebert & Grouws, 2007). This type of discourse aligns with
Stein’s (2000) Procedures with Connections level of the mathematical task. Hiebert and Grouws (2007) pointed out that attending to concepts can occur in a variety of pedagogical forms, ranging from highly structured, teacher-centered classrooms to loosely structured, student-centered formats. In fact, Hiebert and Grouws (2007) noted studies involving various pedagogical and research formats support the facilitation of conceptual understanding through “explicit attention to conceptual development of mathematics” (Hiebert & Grouws, 2007, p. 384). These studies revealed a recurring link between explicit attention to connections among mathematical ideas, facts, and procedures and increased conceptual learning by students (Gamoran, Hiebert, National Research Council, as cited in Hiebert & Grouws, 2007). Attending to concepts promoted not only conceptual development, but also skill learning (Hiebert & Grouws, 2007). Undoubtedly this practical benefit appealed to teachers; nonetheless, Hiebert and Grouws (2007) noted that attention to conceptual development was not a consistent feature in US mathematics classrooms.

Subsequent studies concurred with those explored by Hiebert and Grouws (2007) in support of attention to mathematical concepts in classroom instruction. Researchers have explored the way student interactions supported attention to math concepts and found connections between attending to concepts and students working at a higher cognitive level as well as attaining higher levels of achievement (Marshall & Horton, 2011; Walshaw, 2017; Yu & Singh, 2018). These studies highlighted how research supports Hiebert’s and Grouws’s (2007) premise of attending to mathematical concepts for maintaining a high level of cognitive demand.

Attending to concepts has occurred through student interactions in the classroom. Walshaw (2017) examined the social aspect of students’ mathematical development and conceptual understanding through the Vygotskian lens. In Walshaw’s (2017) study, the
classroom consisted of female students in an accelerated middle school math course. The teacher organized the class to include peer group work and assumed the role of teacher-as-knowledgeable other. A significant situational activity facilitated by the teacher was the opportunity for students to explain and justify their solutions and thinking to classmates. Students expanded their thinking through interacting with peers by discussing, critiquing, and negotiating ideas and solutions. This fit with the teacher’s goal to guide students to rely on peers rather than her, thereby accessing the social aspect of learning concepts (Walshaw, 2017). In addition, the teacher was careful to use students’ ideas to shape instruction, at times showing the whole class the work of a specific group. When introducing a task, the teacher provided carefully chosen pieces of information and asked questions of students to focus on conceptual knowledge rather than simple memorization. This introduction allows students the capability to take personal responsibility for making sense of mathematics, especially as they move on to peer discussions (Walshaw, 2017). Another strategy used was shaping students mathematical language to be precise, using correct mathematical terms, which connected language to conceptual understanding and the larger world of mathematical rules (Walshaw, 2017). Finally, the teacher pressed students for depth in their explanations, especially for incorrect work. This interaction established the idea that wrong was not a permanent state; instead, it was a step toward conceptual understanding and an opportunity for mathematical development. In a group interview of four students, students expressed that they felt both challenged and supported in the class, and found the challenge enjoyable (Walshaw, 2017).

Yu and Singh (2018) observed a strong connection between attending to mathematical concepts and student achievement. In Yu and Singh’s (2018) study, teachers self-reported their degree of emphasis on procedural fluency and conceptual focus during instruction, and compared
teacher survey results to student achievement data. The researchers and teachers alike recognized the need for procedural fluency, but not all teachers placed a strong emphasis on conceptual understanding. Those students whose teachers focused more strongly on conceptual understanding showed higher achievement on a standardized test than those whose teachers focused on procedural fluency alone (Yu & Sing, 2018). When teachers reportedly implemented instruction that had a focus on attending to mathematical concepts, the result was instruction that involved higher-order thinking skills and effective communication. This instructional style led to more in-depth learning than that of standalone procedural fluency, and it became apparent in the achievement test results (Yu & Sing, 2018).

Similar to Yu and Singh’s (2018) findings supporting the value of attending to math concepts, Marshall and Horton (2011) previously found that student exploration of concepts in both math and science classes corresponded to students working at a higher cognitive level. Using data from 102 classroom observations of middle school math and science teachers, Marshall and Horton (2011) sought to understand the connection between the order of phases in an inquiry lesson and the cognitive level at which students were working and learning. Also, Marshall and Horton (2011) explored the time spent on various phases of an inquiry learning process and the resulting cognitive level of students. The inquiry learning process was comprised of four phases: Engage, Explore, Explain, and Extend. Marshall and Horton’s (2011) study focused on the Explain and Explore phases as students investigated math concepts. In the study, students were involved at a higher cognitive level when teachers began a lesson by having students explore concepts with minimal teacher guidance as compared to beginning a lesson with direct instruction. The researchers also found that a higher cognitive level was more likely to result as more time was spent exploring concepts (Marshall & Horton, 2011). Notably, the
difference in cognitive level was even more significant for math teachers than for science teachers. The amount of time dedicated to the lowest cognitive level increased as teachers increased their use of direct instruction to teach computational fluency without regard for conceptual understanding (Marshall & Horton, 2011). Thus, the study illustrated that allowing exploration of concepts played an essential role in maintaining a high level of cognitive demand.

**Struggle with important mathematics.** The second feature of teaching to promote conceptual understanding noted by Hiebert and Grouws (2007) was to have students struggle with important mathematical ideas. By struggle, Hiebert and Grouws (2007) meant students should, “expend effort to make sense of mathematics, to figure something out that is not immediately apparent. (p. 389).” Hiebert and Grouws (2007) summarized Polya (1957) and Dewey (1910), pointing out the value of struggle in experiencing the nature of mathematics and developing an understanding of mathematical concepts. By relating struggle to Vygotsky’s (1978) ZPD, Hiebert and Grouws (2007) indicated that it was necessary to have students struggle through “appropriate problematic situations” (p. 388) so that the goals for students existed in a space in which the struggle students experienced was productive rather than unnecessarily frustrating or fruitless. The researchers explained that struggle restructured understanding and mental connections when new information was not easily assimilated and could even result in a more effective transfer of information to novel tasks, according to Bjork (as cited in Hiebert & Grouws, 2007). Hiebert and Grouws also noted several studies which supported the idea that having students work to solve problems through mathematical tasks with high cognitive demands lead to greater conceptual understanding (Capon & Kuhn, Carpenter et al., Silver & Stein, Stein & Lane, as cited in Hiebert & Grouws, 2007).
Researchers noted how teachers can facilitate struggle with important mathematics. Teacher responses to student struggles played an important role in providing access to essential elements of struggle and maintaining cognitive demand of the task (Barlow et al., 2018; Betts & Rosenberg, 2016; Warshauer, 2015a; Westerman & Rummel, 2012; Zeybek, 2015). The teacher’s method of presenting a task also impacted the resulting level of struggle and cognitive demand (Cheeseman et al., 2016).

With struggle comes the potential for frustration, and teacher support becomes imperative as students learn to persist in struggling with important mathematics. Betts and Rosenberg (2016) noted that regulating frustration was an imperative goal in education, and developing the ability to support students through frustration was a significant challenge for teachers. The researchers focused on productive struggle as a central component of problem-solving ability, and the teachers involved learned to adjust their strategies to better support student struggle (Betts & Rosenberg, 2016). In the study, the teachers initially believed that math teachers were successful when they made learning simple and adapted word problems by reducing the level of cognitive demand (Betts & Rosenberg, 2016). Difficult problems were initially rejected; however, the teachers agreed to try challenging math tasks with their elementary students to explore student struggle. Equipped with a few basic scaffolds, the teachers had students begin work on the first task. The teachers’ initial beliefs were challenged when students happily worked on the task and were willing to struggle when their first solution attempts were incorrect. Not all students reached a correct solution, but due to the level of student engagement in practicing their problem-solving skills, teachers identified the activity as a success in debriefing discussion (Betts & Rosenberg, 2016). This example is illustrative of Kapur’s (2016) concepts
of productive failure and productive success—not all students arrived at a correct solution, but all students were deeply engaged in problem-solving and conceptual development.

Warshauer (2015a) determined that the teacher’s response to student struggles could maintain or reduce the level of cognitive demand of a task, thereby determining whether or not students actually struggled with important mathematics. Warshauer (2015a) devised a Teacher Response Continuum consisting of telling, directed guidance, probing guidance, and affordance.

The first two response types could reduce the cognitive demand of a task (Warshauer, 2015a). Telling was most likely to reduce the cognitive demand of a task, while affordance was likely to maintain the level of demand. Betts and Rosenberg (2016) observed that telling and directed guidance were typical initial instincts of many teachers. Telling often reduced the task to a procedure without connection task when the teacher provided the student with sufficient information to move easily past the struggle without building on student thinking (Warshauer, 2015a). Directed guidance guided the student’s thinking toward a specific solution path, using procedures to deflect struggle, resulting in answers based on the teacher’s thinking instead of the student’s thinking. By reducing the reliance on the student’s thinking, the cognitive demand was reduced (Warshauer, 2015a).

The second two response types were more appropriate for maintaining a high level of cognitive demand (Warshauer, 2015a). Probing guidance addressed student’s struggles by making student thinking visible. The student was asked to explain, justify, or clarify their thinking and consider open-ended questions. In this way, guidance was provided without reducing the cognitive demand of the task (Warshauer, 2015a). In Betts & Rosenberg’s (2016) study, teachers learned to press students to justify their answers rather than merely telling students if they were right or wrong. This technique is an example of probing guidance. Finally,
an affordance response involved limited intervention by the teacher while allowing students to continue to think and build on their ideas. In this response, it was essential to provide motivation and time for students to continue engaging in the task while allowing ample time for students to think and process (Warshauer, 2015a). Betts & Rosenberg (2016) observed that teachers could learn to resist telling students what to do, and instead praise students’ effort to support students’ struggle.

Warshauer (2015a) determined that to maintain the cognitive demand of the task and elicit productive struggle, teachers needed to build on students’ thinking through probing guidance and affordance response strategies, rather than directing students to specific procedures or the teacher’s thinking. Similarly, the teachers involved in Betts & Rosenberg’s (2016) study determined that to facilitate productive struggle, they should focus on the problem-solving process rather than the attainment of correct answers. If students were not problem-solving, they were not engaging in productive struggle (Betts & Rosenberg, 2016).

Zeybek (2016), in the study of PSTs, recognized Warshauer’s (2015a) teacher response strategies and their potential to foster or derail productive struggle. In instances when PSTs struggled to get started and began asking questions, the teacher tended to use telling or directed guidance (Zeybek, 2016). In one instance, the teacher directly answered the PSTs question (i.e., telling). In another, the teacher suggested a strategy the student could try (i.e., directed guidance). Both teacher responses narrowed the possibilities for action by redirecting the PST toward either the teacher’s thinking or a procedure (Zeybek, 2016). Similarly, when PSTs struggled with sense-making and desired additional justification that one group member’s strategy would work, the teacher also used directed guidance to suggest a method to prove the strategy had attained a correct answer (Zeybek, 2016). These types of responses could reduce
the level of cognitive demand of the task, but Zeybek (2016) suggested that at times students needed support to get started or explain their work to proceed with the task and reach the intended level of cognitive demand.

At other times in Zeybek’s (2016) study, teacher responses supported students in doing math. When PSTs struggled with the misconception about whether a rectangular cake could also be a square, the teacher used Warshauer’s (2015a) probing guidance in the form of a brief whole-class discussion to probe PSTs thinking about various dimensions for a cake, and if it could be a square (Zeybek, 2016). The probing guidance response served to maintain the level of cognitive demand of the task. When PSTs struggled to carry out a process, the teacher restated what the PSTs were discussing and allowed more time to work on the task (i.e., Warshauer’s (2015a) affordance). This limited type of intervention allowed PSTs the opportunity to build on their ideas and work through the problem without lowering the level of cognitive demand of the task (Zeybek, 2016). Students explored, constructed, and explained their responses, actions described by Stein (2000) as characteristic of doing math.

Cheeseman et al. (2016) explored how the introduction of a task can impact the maintenance of the level of cognitive demand, determining whether students were engaging in struggle with important mathematics. The premise was that to maintain the challenge of the task, the students must be the ones making the decisions about how to solve the problem, not the teacher. Cheeseman et al. (2016) observed that despite training, teachers tended to pre-teach specific methods when presenting the task, lowering the level of challenge. Such pre-teaching should be avoided when it will reduce the level of cognitive demand of the task (Cheeseman et al., 2016).
Cheeseman et al. (2016) pointed out that every introduction to a challenging task does not need to be identical. Sometimes teachers could briefly read through the task and allow a short time for questions before having students begin. Other times, a discussion relating the task to students’ experiences was beneficial. The key was to avoid suggesting methods for how to solve the task. Generally, Cheeseman et al. (2016) advised refraining from telling strategies or procedures for solving in advance. Instead, teachers should clarify the purpose and expectations of the lesson, as well as encouraging students to persist through the “zone of confusion” (Cheeseman et al., 2016, p. 6). This research falls in line with other researched strategies, such as allowing students the opportunity for progressive formalization (NRC, as cited in Choppin et al., 2012). Other useful teacher strategies were connecting the task to students’ experience, using manipulatives, and communicating enthusiasm about the task (Cheeseman et al., 2016).

Researchers revealed that once a task has been initiated, students still needed support to engage in the task in a meaningful way. Scaffolding strategies should be chosen carefully so as not to reduce the level of cognitive demand of the task. Barlow et al. (2018) presented three scaffolding strategies that can give students access to productive struggle with difficult mathematics. These scaffolds were: elicit prior knowledge, delay the question, and introduce a simpler problem. The purpose of these scaffolds was to ensure that students do not simply struggle, but struggle productively (Barlow et al., 2018).

To use the Elicit Prior Knowledge scaffold, teachers asked purposeful, pre-planned questions that bring to light useful mathematical ideas relevant to the task. Questioning provided access to productive struggle by creating a familiar starting point for students as they reasoned through the task. Questions should not be too general, but rather should elicit specific responses related to the task at hand. For example, when teachers asked students what they knew about
area (i.e., general concept), this was less supportive than asking students to explain an answer they have found for the specific area problem they are working on (Barlow et al., 2018).

To use the Delay the Question scaffold, the teacher first presented the problem scenario without mentioning the task’s question. This introduction put students in a position to make sense of the mathematics in the context of the problem before focusing on the end question. Then, after students have faced the full problem, they will have already begun to explore and make sense of the mathematical relationships necessary to answer the question. This scaffold made the full problem more accessible (Barlow et al., 2018), and aligned with the idea of progressive formalization (NRC, as cited in Choppin et al., 2012).

Barlow et al.’s (2018) final scaffold was to Introduce a Simpler Problem. To use this scaffold, teachers posed a less complex problem that was tied to the key understandings of the primary problem. After students made sense of the mathematical ideas in this simpler problem, they had greater access to the primary problem. Note that the simpler problem did not replace the primary problem; it merely existed to provide access so that all students could engage in the primary problem (Barlow et al., 2018). This scaffold had similarities to Warshauer’s (2015a) directed guidance response but had value because it provided students an entry point into the task. Without an entry point, the task may not be within the ZPD (Vygotsky, 1978) or present and achievable challenge (Willis, 2010).

In all cases, researchers intended for these scaffolds to be temporary support on a student’s path to struggling with important mathematics. Barlow et al. (2018) pointed out the temporary nature of scaffolds. After consistent practice with challenging tasks and productive struggle, students developed a confident ability to reason through problems without prompting.
When this occurred, teachers removed the scaffolds by ceasing to prompt students with prior knowledge supports, delayed questions, and simpler problems (Barlow et al., 2018).

Westermann and Rummell (2012) examined another scaffolding strategy. While Barlow et al.’s (2018) scaffolding directed students toward mathematical content, Westermann and Rummell’s (2012) scaffolding strategy operated independently of mathematical content. These researchers developed and studied the Think, Ask, Understand scaffolding strategy. This type of scaffolding supported productive student struggle by providing a script for student interactions during the initial phase of a problem-solving task before direct instruction had taken place (Westermann & Rummell, 2012). Students were presented with a challenging problem, then worked in pairs, taking turns adopting the roles of questioner and thinker. The thinker verbalized thinking processes, while the questioner posed questions in cases where the thinker was using an incorrect solution strategy or was not understood by the questioner. These roles increased the likelihood of students using metacognitive processes during problem-solving, and students who used the script performed significantly better on post-tests than those who did not (Westermann & Rummell, 2012). The study lasted for several weeks, and as time passed, learning outcomes as measured by student achievement post-tests increased for those using the role script scaffold. Students internalized the script and allowed it to pave the way for increased learning. Because this scaffold only supported students’ interactions, not math content, leaving this scaffold in place did not deny the opportunity to struggle with important mathematics on a deep level. Instead, the likelihood of meaningful mathematical struggle was increased by the continued use of the role script scaffold (Westermann & Rummell, 2012).
Classroom Norms

In comparison to traditional teacher-led instruction, students and teachers have needed to rethink their roles and how they defined success within a classroom where productive struggle was the norm (NCTM, 2014). Smith (as cited in NCTM, 2014), suggested several shifts in defining success for students as well as defining the teacher’s role. Students might have been accustomed to presenting correct solutions as a measure of success; however, in a classroom that fostered productive struggle, the reasoning behind the solution was of utmost importance. Further, students may not have felt that success and frustration go hand in hand; conversely, in a productive struggle classroom, perseverance through frustration was expected and anticipated (Smith, as cited in NCTM, 2014). The teacher’s role in supporting students also had a subtle shift. Instead of asking leading questions directing students to a pre-set solution path, teachers supported productive struggle when their questions focus on the students’ reasoning (Smith, as cited in NCTM, 2014). Further, teachers moved beyond checking for correct answers and instead insist that students follow through to explain and justify their solutions (Smith, as cited in NCTM, 2014). As the classroom leader, the teacher communicated to students that struggle was vital to their learning (NCTM, 2014). Class culture and norms played a chief role in students’ motivation to engage in productive struggle by leveraging peer support and teacher-student relationships (Livy et al., 2018; Townsend et al., 2018).

Peer support was a main component in a classroom culture that encouraged productive struggle. To better reflect on their understanding, students respectfully questioned and responded to their peers as the class reasoned through rich tasks (Smith, as cited in NCTM, 2014). Teachers utilized peer support by establishing group norms such as, “You have the right to ask anyone in your group for help,” and “You have the duty to assist anyone who asks for
help” (Townsend et al., 2018, p. 222). Supportive peer groups minimized perceived ability differences, resulting in students persevering more readily, with less anxiety, and more willingness to participate in group discussions (Townsend et al., 2018).

Teacher-student relationships were essential aspects of classroom culture to allow productive struggle to occur. The research suggested that teachers construct the framework for a mindset conducive to learning and growing even through frustration-inducing challenges (Dweck, 2008; NCTM, 2014; Townsend et al., 2018). Dweck (2008) referred to this mindset as a growth mindset—the mindset that intelligence was not innate, but instead can be developed through effort. NCTM (2014) stated that “Teachers must acknowledge and value students for their perseverance and effort in reasoning and sense-making in mathematics” (p. 50) instead of praising apparent intelligence or correct answers. Teachers should be available for support and provide specific feedback that will result in growth (NCTM, 2014). For example, to encourage engagement and persistence, a teacher should praise effort, perseverance, and willingness to ask questions, while offering feedback on how to improve on habits of doing math, such as using precise explanations (NCTM, 2014). Townsend et al. (2018) observed that caring for students by being available for assistance and taking an interest in their life experiences and opinions laid a foundation for motivation to learn math. Teachers could also foster productive struggle through encouragement in the form of meaningful public praise and assigned competence because these strategies minimized perceived ability differences. Additionally, teachers could emphasize norms for respect, such as disagreeing with ideas rather than people. Such norms fostered a classroom climate that affords engagement in challenging math tasks (Townsend et al., 2018).
Livy et al. (2018) suggested that during class discussion of a challenging math task, students must be able to present their ideas in an environment where teachers and peers respond in a manner that does not make judgments about students’ abilities. Instead, teachers and students posed questions and responses with sensitivity and respect, maintaining a focus on student thinking. This practice promoted a growth mindset as well as productive struggle (Livy et al., 2018).

Grouping of students was another classroom norm that impacted students’ opportunity for productive struggle. Wiedmann, Leach, Rummell, and Wiley (2012) found that the makeup of small student groups impacted the number and quality of possible solutions that the groups generated during a challenging math task. The researchers found that mixed ability groups had more solution attempts and produced more varied representations of solution paths than students grouped with their like-ability peers (Wiedmann et al., 2012). The concept applied to both high-ability and low-ability groups—the mixed-ability groups outperformed both in terms of generating solutions and post-test results. The pattern suggested that developing the classroom norm of collaboration between various abilities of students in small groups supported productive struggle (Wiedmann et al., 2012).

Sullivan and Mornane (2013) gleaned several important classroom culture fundamentals from a study involving student interviews following a challenging math task lesson. First, some students noted that they felt more comfortable persisting in the task without fear of failure when the teacher told the class that multiple solution paths were possible. They were willing to show their work to peers, even if it meant risking making an error publicly. Second, the students stated that the teacher was the most influential factor in their decision to try hard on challenging tasks. Students recognized the constructive nature of the teacher’s feedback, noting that it allowed them
to know how to improve for the next task (Sullivan & Mornane, 2012). Also, the constructive feedback built trust; students felt comfortable enough in the teacher/student relationship to persist in the task with the knowledge that the teacher was there for support if the struggle became too great. They were not afraid to make mistakes in front of their peers or their teacher (Sullivan & Mornane, 2012). These factors assisted students’ ability to persist through a challenging task, providing the opportunity for productive struggle (Sullivan & Mornane, 2012).

**Teacher Preparation to Support Student Struggle**

The NCTM (2014) suggested that U.S. mathematics instruction rarely required students to think and reason with mathematical ideas. In comparison to other high-achieving nations, students in the U.S. did not face an expectation to grapple with mathematical concepts with the same rigor (Banilower et al., as cited in NCTM, 2014). When students struggled, U.S. teachers tended to help quickly instead of allowing students to experience frustration (NCTM, 2014). Productive struggle in the U.S. classroom had been a deviation from more traditional direct instruction approaches to math teaching, so incorporating struggle was a skill that both novice and veteran teachers must develop (Barlow et al., 2018; Betts & Rosenberg, 2016). During lesson planning, teachers should consider potential student struggles and misconceptions, and plan for supports that maintain the intended conceptual rigor of the task (NCTM, 2014).

Betts and Rosenberg (2016) asserted that both the planning process and the teacher mindset equipped teachers to support students’ productive struggle. The productive struggle pedagogy Betts and Rosenberg (2016) developed in their action research began with the idea that teachers should be open-minded about with students can do. Open-mindedness allowed teachers to identify student strengths and build on those to tackle challenging problems. The next facet of their pedagogy was a focus on problem-solving processes rather than answers. With practice,
teachers developed their ability to provide the right scaffolds to move students along without
telling them how to do a problem (Betts & Rosenberg, 2016). This focus on process related to
Smith’s emphasis on student reasoning and justifying solutions (as cited in NCTM, 2014).
Finally, in Betts & Rosenberg’s (2016) pedagogy, teachers considered the diverse learning needs
of the classroom. The teacher must have decided how much assistance to provide to the entire
class, as well as to individuals. For example, was it always necessary to provide a pre-organized
recording sheet? Organization was a problem-solving skill that students needed to practice in
some tasks. Other times, it was best to help the students focus on other aspects of the task by
providing organizational charts. Through their professional learning journey, these researchers
determined that their initial efforts to support student struggle further developed their
professional ability to do so (Betts & Rosenberg, 2016). The more knowledge they had of their
students’ problem-solving abilities, the better they could select and provide scaffolds and
minimal guidance to maintain the level of cognitive demand and allow students to productively
struggle. Their professional relationships with students guided their decision making regarding
what support to provide (Betts & Rosenberg, 2016), highlighting the importance of teacher-
student relationships in motivating students toward productive struggle (Livy et al., 2018;
Townsend et al., 2018).

Some traditional teaching skills supported students in productive struggle. Teachers used
the concept of scaffolding previously in education, and teachers were already familiar with the
idea of providing temporary support so that learners could complete a task they may not be able
to complete otherwise (Barlow et al., 2018). Barlow et al. (2018) proposed an additional
function for scaffolding—to provide students access to productive struggle opportunities. Thus,
a skill that teachers could expand and refine a skill they already possessed to engage students in
productive struggle. Scaffolding was closely tied to Warshauer’s (2015a) directed guidance response, which often provided an entry point to a challenging math task in a moment when students might otherwise abandon their efforts. In one example, Barlow et al. (2018) described how a teacher accessed students’ prior knowledge as a scaffold by asking them to draw a rectangle with a known area on grid paper before asking them to find the area of an irregular figure (Barlow et al., 2018). With a previous group, this scaffold was not in place. Those students manipulated area formulas incorrectly, and their struggle was not productive. However, with the second group that had the scaffold, this mistake did not happen, and they successfully made sense of the mathematics through productive struggle. The teacher did not initially realize the need for the scaffold—it was the teacher’s own experience with the first group that better prepared her to plan for productive struggle for the second group (Barlow et al., 2018). The teacher recognized the opportunity to apply her previously known skill. Teachers possessed skills that applicable to fostering productive struggle, and greater experience utilizing these skills further develops teachers’ ability to implement the skills (Barlow et al., 2018).

In the previous examples, teachers chose intrinsically to develop their skills in providing opportunities for productive struggle through action research (Barlow et al., 2018; Betts & Rosenberg, 2016); however, administrators and other school leaders could initiate professional development that will support teachers as they incorporate challenging math tasks and productive struggle into their classrooms (Sullivan et al., 2015). Sullivan et al. (2015) provided groups of elementary and middle grades teachers with lessons built around challenging tasks as well as two days of focused training on the rationale and strategies for eliciting productive struggle in the lessons. Teachers worked through the lessons during the training. From the participants’ experiences, they drew connections to strategies that school leaders can use for professional
development to move teachers toward being comfortable incorporating challenging tasks and productive struggle in their teaching (Sullivan et al., 2015). Sullivan et al. (2015) assumed that a barrier to implementing challenging tasks was that teachers assume students would be reluctant to participate. Another assumption was that teachers sought out lessons developed by others, not because of lack of ability to develop their own, but lack of time to do so. After participation in the training and teaching the lessons, most teachers disagreed with the idea that this type of lesson was only for the best students, evidence that such training helped overcome the first barrier (Sullivan et al., 2015). In addition, teachers in Sullivan et al.’s (2015) study indicated that they felt the provided lesson was easy enough to use and effective in eliciting student persistence throughout the challenges they faced. The teachers’ responses suggested that providing lessons to teachers supported overcoming the second barrier. When researchers asked teachers if they thought a colleague could effectively utilize the lessons without also having attended professional development, responses were mixed, which suggested that first-hand participation in professional learning was a significant factor (Sullivan et al., 2015).

Conclusion of the Literature Review

Productive struggle has had a significant purpose in student learning of mathematics (Betts & Rosenberg, 2016; Hiebert & Grouws, 2007; NCTM, 2014; Warshauer, 2015a). It has proven challenging for teachers and students alike, so teachers needed to be well equipped to support students in their struggle (Cheeseman et al., 2016; Sullivan et al., 2015). The literature identified types of student struggle as well as strategies for support, along with teacher professional growth pathways toward supporting student struggle (Barlow et al., 2018; Warshauer, 2015a). Productive struggle was underpinned by the concept of doing math (Stein, 2000; Van de Wall, 2016) while employing sense-making as students reason through challenging
math tasks (Moss & Lamberg, 2016; Mueller et al., 2011). An additional dimension which this research aimed to address was students’ perspective on their personal experience with productive struggle. The purpose was to assist teachers in understanding students’ experiences so teachers can better meet students’ learning needs. In chapter 3, the researcher will describe the methodology of the study to gain insight into student perspectives on productive struggle.
CHAPTER THREE: Research Methodology

The purpose of this study was to delve into the experiences of students as they persisted with productive struggle on challenging math tasks. Existing researchers suggested that engaging in productive struggle facilitated lasting learning (Hiebert & Grouws, 2007; Stein, 2000), and other researchers emphasized teacher actions that maintain the level of cognitive demand needed for productive struggle (Barlow et al., 2018; Cheeseman et al., 2016; Warshauer, 2015a). Through a qualitative methodology, the researcher developed insight into students’ thoughts, feelings, challenges, and needs as the students participated in challenging math tasks requiring productive struggle in a classroom setting where interactions maintained the level of cognitive demand. In any class, students had varying levels of comfort with challenging inquiry-based math tasks. The aim was to understand the lived experience of students as they practiced productive struggle and understand how productive struggle impacted mastery of standards as assessed on TNReady.

Research Questions

The researcher used the following research questions in this phenomenographic study.

Research Question 1: What patterns of productive struggle occur for bubble students as they work on challenging math tasks supporting mastery of instructional objectives in Algebra II and Geometry in one high school in Tennessee?

Research Question 2: What is the relationship between patterns of productive struggle and mastery level shifts for bubble students in Geometry in one high school in Tennessee?

Qualitative Research

Qualitative research was appropriate for understanding how individuals functioned within a particular context, such as a classroom setting, as well as understanding the context from the
perspective of the participants (Hatch, 2002). The researcher utilized videoed observations, semi-structured interviews, and collection of artifacts to gain insight into students’ experience, utilizing the researcher as the data gathering instrument. First, through observation of teacher-student interactions, the researcher documented the actions and attitudes of students in recordings and journal entries. Such participant observations allow the qualitative researcher to discover inductively how the participants understand the context of the setting, herein the context being the experience of working on a challenging math task. Then, the research conducted and recorded student interviews to document students’ perceptions of their productive struggle. In qualitative research, interviews helped the researcher uncover and clarify any initially hidden concepts that participants use to make sense of their worlds (Hatch, 2002). Additionally, the researcher examined artifacts in the form of student work on inquiry-based activities in lab books to uncover patterns of productive struggle. In qualitative research, artifacts can give insight into participants’ thoughts and actions—here, specifically the patterns of thinking and struggling productively through the math tasks (Hatch, 2002). Finally, the researcher examined TNReady data for geometry students to see which bubble students advanced to the next level of mastery, and reviewed previous data to check for distinguishable differences between those who advance and those who do not. Achievement records such as test score data were another useful artifact in qualitative research (Hatch, 2002).

Research Design

The researcher used a qualitative research design, specifically phenomenography, to answer the research questions. The study aimed to understand the lived experiences of students as they experience productive struggle and understand how productive struggle impacts mastery of standards as assessed on TNReady. Phenomenography allowed the researcher to gain insight
The purpose of the study was to explore and understand students’ personal experiences with productive struggle. The first research question was: What patterns of productive struggle occur for *bubble* students as they work on challenging math tasks supporting mastery of instructional objectives in Algebra II and Geometry in one high school in Tennessee? To gain insight into students’ experiences, the researcher used a combination of recorded observations, interviews, and artifacts related to students’ work on challenging math tasks. As students set to work on challenging math tasks during class time, the researcher recorded conversations with the selected participants to identify patterns of productive struggle through observation. Next, the researcher created student interview questions based on themes that occurred during class time observations to understand student perceptions better and identify trends. Finally, the researcher examined student work samples from the tasks for themes matching the emerging themes from observations and interviews. The combination of observations, interviews, and student work samples provided data triangulation.

The second research question was: What is the relationship between patterns of productive struggle and mastery level shifts for *bubble* students in Geometry in one high school in Tennessee? The researcher used Fall 2019 TNReady data to determine which bubble students advanced the next level of mastery. Then, the researcher examined experiences of the advancing students and non-advancing students for trends unique to each group.

**Data Collection**

Data collection included observations, interviews, and student work samples. Each of these types of data offered a window into the students’ lived experiences during productive
struggle. Throughout the process, the researcher needed to bracket her expectations and assumptions to be fully open to ascertain the students’ true perspectives (Trigwell, 2000). The researcher obtained permission from the Carson-Newman University Institutional Review Board to proceed with the study and begin data collection. The researcher selected participants from the class roster based on test scores falling within seven raw score points for the next highest achievement level on the previous year’s TNReady End-of-Course assessment. All students in the classroom completed the challenging math tasks, but the researcher recorded and interviewed only those meeting the bubble student criteria. Other students were possibly on the audio recordings made during class. For this reason, the researcher obtained permission from the parents of all students through a consent form (See Appendix A). The data collected revolved around students’ work on two challenging math tasks taken from the NCTM’s 2012 publication *Implementing the Common Core State Standards through Mathematical Problem Solving* (See Appendix B). The first task began as follows:

The perimeter of a rectangle is represented by $4x + 24$. In response to the question, “What is a possible representation of the area of this rectangle?” Mark says, “$x^2 + 36$,” and Sarah says, “No, $x^2 + 12x + 36$.” What assumptions are both Mark and Sarah making about the rectangle? Comment on their answers. Other students are also discussing the problem. Alex says that the area is $24x$. Anna says that she thinks the area is $x^2 + 12x + 27$. Where do you think they are getting these answers? Can they all be correct? (NCTM, 2012, p. 8)

In addition to these questions, the first task had an additional question, as suggested in the NCTM (2012) publication: Are there other correct areas for a rectangle with perimeter $4x + 24$? Explain.
The second task had two parts, adapted from the same publication. The first part was “Find two numbers whose sum is 4 and whose product is 5 (NCTM, 2012, p. 79)”. The researcher slightly modified the second part of the task from the original text for clarity:

[In part A, students created a quadratic equation] (a) Express the solutions to the quadratic equation you found in [complex] form. (b) [The task] asked you to find two numbers whose sum is 4 and whose product is 5. Do these numbers satisfy these requirements? Why or why not? (NCTM, 2012, p. 79).

The researcher gave the first task to both Algebra II and Geometry students, and gave the second task to Algebra II students only.

First, the researcher made recordings as students worked on math tasks selected from those developed by NCTM and considered to be high level, doing math tasks. The researcher used the three-phase model to structure class time: independent think time, group time, class discussion (University of Pittsburg Institute for Learning, as cited in Tennessee Department of Education, 2012). The researcher interacted with students in her typical capacity as the classroom teacher, the only difference being recording conversations with participants. The teacher used appropriate strategies—probing guidance and affordance (Warshauer, 2015a)—to maintain the level of cognitive demand of the task so that students would have the opportunity to engage in productive struggle. Questioning techniques elicited evidence of student thinking as they struggled through the task. The researcher asked students to explain, justify, or clarify their thinking, as described by Warshauer (2015a). The researcher recorded student-teacher conversations during the math task for later transcription and analysis.

Next, the researcher conducted small-group student interviews to further dissect their experiences during class time, as interpreted in the recordings. The researcher took students
Aside in small groups to conduct the interviews at a table in the classroom while another teacher stepped in to assist the other students with an assignment. These interviews were recorded. The interviews were semi-structured and allowed participants to discuss their perception of the meaning of the phenomenon they experienced from a second-order perspective (Marton, 1986). Interviews focused on students’ feelings of the degree of self-efficacy as well as their experiences with the specific student struggle types observed (See Appendix C). Warshauer’s (2015a) struggle types were used in categorizing students’ struggle: struggle to get started, struggle to carry out a process, struggle to explain or make sense, and struggle to express misconceptions or errors. During the interviews after the task, the researcher asked about students’ feelings at the beginning, middle, and end of the challenging math tasks. Through the interview questions, the researcher explored different aspects of the students’ experiences as thoroughly as possible, as required by sound phenomenographic research (Trigwell, 2000). The interview time was also a chance for the researcher to perform member checks to ensure the accuracy of the researcher’s understanding of the students’ experiences (Ary, Jacobs, Sorenson, & Walker, 2013). The researcher performed member checks throughout interviews by asking participants for clarification and repeating answers back to participants to ensure understanding of their responses. This summarizing and restating ensured there were no misconceptions about responses. After the interviews, the researcher performed member checks for the recorded classroom interactions by reviewing and summarizing the recorded conversations and asking for clarification on statements to ensure understanding.

Finally, the researcher collected and analyzed student work samples from the tasks. In this part of the data collection, the researcher sought to confirm through triangulation the conclusions drawn from the recordings and interviews (Ary et al., 2013). The data collection
process took place over three weeks in the middle of the second 9-weeks of the school year. The researcher also used member checks in the interpretation of student work samples. The researcher asked students to elaborate on and clarify their thinking during and after the work.

Peer debriefing confirmed the validity of conclusions (Ary et al., 2013). Professional colleagues not directly involved in the study agreed to review raw data as well as the researcher’s interpretation to determine the reasonableness of the interpretation given the evidence. Peers reviewed transcripts, student work samples, and section drafts.

**Coding Process**

The analysis focused on the emotions and struggles students experienced during the tasks and how they conceptualized the phenomenon, participants typically understood each phenomenon in a finite number of ways (Marton, 1986). To answer the first research question, the researcher transcribed and coded observation recordings to identify the students’ experiences and perceptions in connection to the various types of productive struggle. Similar to a process described by Marton (1986), data analysis included reducing the data, finding relevant statements, and constructing categories. The researcher grouped statements in transcripts by meaning into themes and created categories from these themes. The researcher organized the categories into an outcome space showing the relationship between topics (Marton, 1986). Finally, the researcher observed evidence of student thinking in student work samples to confirm the thematic clusters in the observations and interviews. The work samples allowed the researcher to confirm the categories and outcome space detailing the various experiences of students engaging in productive struggle.

To answer the second research question, the researcher used TNReady data to identify which bubble students advanced to the next level of mastery and which did not. Then, the
researcher compared the experiences of the two groups of students for similarities and differences to reveal trends for either group. The same categories and themes applied in the second research question as to the first.

**Study Participants and Setting**

The researcher conducted this study in a rural high school in northeast Tennessee. The school district is large, consisting of four high schools with 500 – 1000 students each, as well as seven middle schools and eleven elementary schools. For this study, the researcher selected a convenience sample of students in Algebra II and Geometry from her class roster, at a rural high school with about 800 students. There was little ethnic diversity at the school, with nearly all students being Caucasian. There was some socioeconomic diversity, but the school was classified as a Title I school.

Specifically, the researcher selected bubble students as participants—those students just on the cusp of scoring on the next achievement level on the previous year’s TNReady End-of-Course state assessment. Using bubble students as participants allowed observation of students on various levels. It also allowed observations about whether these students advanced to the next level of mastery as measured by the state test at the end of the course, drawing comparisons between the experiences of those who advanced and those who did not. The researcher also served as the classroom teacher. The researcher implemented strategies to facilitate productive struggle and maintain the level of cognitive demand of tasks as described by Warshauer (2015a).

**Ethical Considerations**

Qualitative research involving human participants required a high standard of ethics. To maintain this standard, the researcher first obtained permission from Carson-Newman University’s Institutional Review Board. The researcher contacted the parents of all students on
her roster regarding the study. Students worked in groups, so the researcher obtained permission for recordings and interviews from all parents of students grouped with the participants because all group members could potentially be heard in the audio recordings. The researcher assigned pseudonyms to all students involved. In the final report of the study, the researcher presented all methods and data accurately and without bias.

**Summary of Research Methodology**

The purpose of the study was to examine students’ experiences with productive struggle as the experiences related to mastery of standards for bubble students. The collection of data was qualitative, except for TNReady data used to identify the level on which the students performed. The researcher used observations, interviews, and student work samples for qualitative data related to students’ productive struggle experiences and used Geometry TNReady data for quantitative data related to bubble students’ advancement to the next mastery level. Chapter 3 included an introduction with a description of research questions, population and sample description, description of research procedures, and data analysis.
CHAPTER FOUR: Presentation of Findings

The purpose of this study was to understand the lived experience of students as they practiced productive struggle and understand how productive struggle impacted mastery of standards as assessed on TNReady. To understand these experiences and impacts, the researcher gathered data from several sources. Data sources were observations, interviews, and student work samples. The observations were in the form of audio recordings of teacher-student interactions as well as peer interactions throughout work on the task. Interviews were held in small groups and consisted of open-ended questions with a semi-structured format allowing for clarification and detailed responses. Four Algebra 2 students and nine Geometry students participated. The researcher organized interview questions into two sections. Students first answered questions about their thoughts and feelings as they worked on the beginning, middle, and end of the task. Questions prompted students to specifically discuss thoughts and feelings during each phase. In the second section, students then answered questions about what they perceived as the most challenging and least challenging parts of the task. The most or least difficult part of the task could be a specific question from the task or an overarching skill or concept required throughout the task. The interviews lasted an average of 20 minutes. Student work samples consisted of the completed handout with the task questions. This chapter presented the data as it related to the two research questions. The researcher used data from all students to answer the first research question and used data from Geometry students only to answer the second research question.

Selection of Participants

Participants included four Algebra II students and nine Geometry students. All participants met the criteria to be identified as bubble students, scoring just on the cusp of the
next achievement level within the four achievement levels identified by TNReady data. The research selected students from her class roster. The participants represented a variety of achievement levels.

**Participant Demographics**

Participants included students in grades 9, 10, and 11. Participants consisted of one Algebra II student on the cusp of scoring level 2, and three Algebra II and five Geometry students on the cusp of scoring level 3, and four geometry students on the cusp of scoring level 4. The researcher asked a larger number of students to participate; final participants were those who turned in parent consent forms and were present on the day of the observations. The Geometry students were in an honors-level class, while the Algebra II students were in a general level class.

**Research Questions**

The research questions guided the researcher to examine the productive struggle experience of students and assess the implication for mastery.

Research Question 1: What patterns of productive struggle occur for *bubble* students as they work on challenging math tasks supporting mastery of instructional objectives in Algebra II and Geometry in one high school in Tennessee?

Research Question 2: What is the relationship between patterns of productive struggle and mastery level shifts for *bubble* students in Geometry in one high school in Tennessee?

**Data Analysis**

The researcher utilized a semi-structured interview approach with five questions focused on students’ feelings throughout the task and students’ perception of the task’s difficulty. The researcher designed all interview questions to help answer the research questions posed in this study. The researcher conducted observations of student-teacher and student-student interactions
as students worked on the task. The researcher recorded and transcribed all interviews and observations for coding. The researcher also used student work samples as an artifact showing the result of students’ productive struggle. The coding process involved open-coding in determining if there were patterns of meaning to the interview data, observation data, and work samples, followed by grouping into broader categories, and finally, into selective codes to represent themes within the participant responses and work. Charts follow to demonstrate this process for research question 1 (see Figures 4.1, 4.2, 4.3, and 4.4). For research question 2, the researcher identified statements and observations as belonging to advancing or non-advancing Geometry students, and comparison charts identify trends in similarities and differences between the groups (see Figures 4.5 and 4.6).
**Figure 4.1 Data Sorted in Levels of Coding for Research Question 1 – Beginning of Task**

<table>
<thead>
<tr>
<th>Interviews</th>
<th>Raw Data</th>
<th>Observations ([Work Samples])</th>
<th>Open Coding</th>
<th>Axial Coding</th>
<th>Selective Coding</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;I felt like I know what to do, but it turns out I didn’t.&quot;</td>
<td>&quot;It's not a square, so it won’t be equal. It said it’s a rectangle.&quot; (A) (several students)</td>
<td>&quot;I'm not sure what to do.&quot; (A) (several students)</td>
<td>Students initially misjudged the difficulty of the task or their own level of understanding.</td>
<td>Despite initial confidence, students struggled with expressing a misconception or error.</td>
<td></td>
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<tr>
<td>&quot;I'm most part of the beginning I knew what I was doing, but then I was completely lost.&quot;</td>
<td>&quot;Isn't the area formula length times width times height?&quot;</td>
<td>&quot;Oh, multiplication. I was doing subtraction.&quot; (A)</td>
<td>Students made mistakes, but correcting those mistakes propelled their work forward.</td>
<td>With an initial sense of confusion, students struggled to get started.</td>
<td></td>
</tr>
<tr>
<td>&quot;I was actually doing the problem wrong.&quot; (A)</td>
<td>&quot;(x² + 12x = 36) (N)&quot; (&quot;Mark thinks if he factored he would get x² = 36.&quot;)</td>
<td>(Student 6 couldn’t accept that the area was not x² = 36, and kept this as a correct answer on their paper. (A))</td>
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<tr>
<td>&quot;At first, I was just completely lost.&quot;</td>
<td>&quot;I'm not sure what to do.&quot; (A) (several students)</td>
<td>&quot;I'm lost trying to figure this out.&quot;</td>
<td>Students initially felt lost and confused.</td>
<td>Students experienced internal tension and negative self-talk.</td>
<td></td>
</tr>
<tr>
<td>&quot;I don’t know how they got the answers they were getting.&quot; (A)</td>
<td>&quot;I'm confused.&quot; (Some students had nothing written on their papers and expressed confusion to the teacher.)</td>
<td></td>
<td>Students felt anxious, uneasy, frustrated, and overwhelmed.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;My first thought was, you can’t find it out.&quot; (A)</td>
<td>&quot;Nevermind, I was stupid.&quot;</td>
<td></td>
<td>Students admitted confusion but pressed through it to attempt work.</td>
<td>Students experienced struggle to carry out a process and struggle to make sense of their work.</td>
<td></td>
</tr>
<tr>
<td>&quot;It was going to be hard.&quot; (N)</td>
<td>&quot;I am seeing if I can figure out a way to get these two and figure out what they're actually thinking, but how can I get dimensions from this?&quot;</td>
<td>&quot;So, yeah, am I multiplying this?...How am I going to put these two in here?&quot;</td>
<td>When asked to explain their work, their questions surfaced.</td>
<td>Students pragmatically worked toward a solution, pushing through struggles to carry out a process and make sense of their work.</td>
<td></td>
</tr>
<tr>
<td>&quot;(It) was confusing because it was the very first one.&quot; (N)</td>
<td>&quot;I was going to do that, but I didn’t think anything could go into 27.&quot;</td>
<td>&quot;I don’t know how you could know the length in that.&quot; (A) (One student had correct work, then erased it.)</td>
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<tr>
<td>&quot;I say the same thing, but more anxious, too.&quot;</td>
<td>&quot;[I am seeing] if I can figure out a way to get these two and figure out what they're actually thinking, but how can I get dimensions from this?&quot;</td>
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<tr>
<td>&quot;It was overwhelming at first.&quot;</td>
<td>&quot;I don’t know how you could know the length in that.&quot; (A) (One student had correct work, then erased it.)</td>
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<tr>
<td>&quot;I was feeling pretty overwhelmed.&quot; (N)</td>
<td>&quot;I’m confused.&quot;</td>
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<tr>
<td>&quot;Frustrated. I had to go back and do it all over again.&quot;</td>
<td>&quot;I’m lost trying to figure this out.&quot;</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>&quot;I felt stupid.&quot; (A, N)</td>
<td>&quot;I feel stupid.&quot;</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;At first I completely forgot about factoring.&quot; (A)</td>
<td>&quot;I am seeing if I can figure out a way to get these two and figure out what they're actually thinking, but how can I get dimensions from this?&quot;</td>
<td>&quot;[I am seeing] if I can figure out a way to get these two and figure out what they're actually thinking, but how can I get dimensions from this?&quot;</td>
<td></td>
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</tr>
<tr>
<td>&quot;I didn’t know what to do. So, I immediately thought, I’m doing guess and check.&quot; (A)</td>
<td>&quot;So, yeah, am I multiplying this?...How am I going to put these two in here?&quot;</td>
<td>&quot;[I am seeing] if I can figure out a way to get these two and figure out what they're actually thinking, but how can I get dimensions from this?&quot;</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>&quot;I didn’t think any of it was easy at the beginning. Once I finally grasped how to do the quadratics, it was pretty easy.&quot; (A)</td>
<td>&quot;I was going to do that, but I didn’t think anything could go into 27.&quot;</td>
<td>&quot;I don’t know how you could know the length in that.&quot; (A) (One student had correct work, then erased it.)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;I needed to know what I’m working with to know how to start.&quot; (A)</td>
<td>&quot;I am seeing if I can figure out a way to get these two and figure out what they're actually thinking, but how can I get dimensions from this?&quot;</td>
<td>&quot;[I am seeing] if I can figure out a way to get these two and figure out what they're actually thinking, but how can I get dimensions from this?&quot;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;In the beginning, [part] A was probably the easiest.&quot; (A)</td>
<td>&quot;I’m trying to figure out how Sarah got this.&quot; (A)</td>
<td>&quot;I’m on the process of working that out.&quot; (A)</td>
<td>Students pragmatically worked toward a solution, pushing through struggles to carry out a process and make sense of their work.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>&quot;In the beginning, I was actually quite confident.&quot;</td>
<td>&quot;I’m trying to figure out possible lengths and widths for the sides.&quot; (A)</td>
<td>&quot;I think—let me figure this out. So the reason I have this here, the dividing by 4, if I get this, it can go into all of these.&quot; (Students worked independently, without prompting. They explained what they were working on and showed their thinking on their paper.)</td>
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</tr>
</tbody>
</table>

(A) indicates an advancing bubble student made this statement.
(N) indicates a non-advancing bubble student made this statement.
**Figure 4.2 Data Sorted in Levels of Coding for Research Question 1 – Middle of Task**

<table>
<thead>
<tr>
<th>Interviews</th>
<th>Raw Data</th>
</tr>
</thead>
</table>
| "Middle, not necessarily stressful, but something like that." (A) | "Did they add or do they multiply?" "For area of perimeter?" "For area."
| "Confusing." (N) | "Do you just do the quadratic equation from here?" "That's what I'm trying to figure out."
| "I felt like I knew what was going on, but then I was like..." (A) | "I don't know which method I'm supposed to use."
| "The middle was definitely the hardest for me. It was really confusing." (A) | "I'm not sure how..." (Several students) (A)
| "I forgot about factoring again." (A) | "So now, what in the world?"
| "Yeah, the middle was also the hardest part for me because we had to do a lot of thinking and it was also the productive struggle, you make us do." (N) | "I'm looking at this, and wondering, how...?" (A)
| | "I'm just not sure what to do." (Several students) (A)
| | "There is where I don't really understand."
| | "It took me longer to figure out how to factor because the snowflake factoring method doesn't really work for me." (A)
| | (Some students had answers but lacked a full written explanation.)
| | (Alex thinks all sides equal 6 cause 6*4=24) Question mark indicates confusion.) (N)
| | "I thought I knew what to do, I just had to remember what are some math rules, how am I supposed to do this correctly? I just needed to make sure I didn't do anything wrong." (A)
| | "I thought I knew what to do... I just wasn't sure if it was exactly the right thing to do, but I just went with it and hoped for the best." (A)
| | "The most challenging, was just like having to figure out some problems... there's always this one part, we don't know what to do next. We had an idea, but we had no idea if that was actually the right direction." (N)
| | "These two groups are different answers. I made assumptions and they're both... One's a square and one's a rectangle." (N)
| | "If you do put it on a rectangle, would you put like x+1 right here and x + 0 over here?" "I'm trying. I just get there eventually. So I'm going to get an x (put an x on all four sides), yeah, because a rectangle could be a square." (N)
| | "Something will be multiplied to get 36 w/o a variable. Possible answers could be 4*9, 6*6, 8*4.5"
| | "If I change this number, this number also changes, so I'm looking at [factors] of 12 and 13. What numbers go into both?" (A)
| | "So you have y* and y* so that's already your a and b. So you have to set the equation equal to zero instead of five (continues to correctly explain how to use the quadratic formula to solve)"
| | "I don't want this there. I added them up instead of multiplying. Instead of 54, I get 94, which is the same as this." (N)
| | "[y+3)(y+3) = x² + xy + 9x + 9 + xy + 3x + 27 + x² + 12x + 27 (Mary had correct factorizations as well."
| | "When I realized how they got their answers, it was easy to do." (A)
| | "Then whenever I got to part B, I was like, well A is the only answer, and it took me a minute to realize it wasn't." (A)
| | "I kept thinking it was going to be the same numbers as the first time, but then I realized that it's completely different things." (A)
| | "I understood more with the diagrams and realising I could just factor those quadratics." (A)
| | "And then I realized two of the equations were quadratics, so I factored them to find length and width." (A)
| | "After we found the quadratic, finding the roots and realizing which one was the right one was easier." (A)
| | "I didn't think anything of it was easy at the beginning. Once I finally grasped how they got the quadratics it was pretty easy." (A)
| | "Actually, nevermind, because this one, one of them wasn't in x. I just know... One of the side lengths isn't going to have an x in it." (N)
| | "So that's right? Yes." "Cool, I did math!" (A)
| | "12 times 12, no, because that'd mean that's not full. Oh, wait a second, two, what about that?" "What about that?" "Right there, yes, yes, because area is length times width and we're looking for 24x, so 12 time 2x is 24x." (N)
| | "He's just multiplying this here and he just gets that. That's what I did." (An incorrect method is written on the student's paper, but he eventually identifies it as incorrect.)
| | "We talked about Mark because I had no clue how Mark would've gotten this unless he's messed something up because the 2x, you can't get that unless you also multiply another number, which would give you an x², an x, and a regular number with no variable. So I was like, he had to mess up somewhere or something." (A)
| | "I did that and then whenever you multiply those together. I think, no wait, because that gives you the—uhh?" (Continues working problem correctly.) (A)

<table>
<thead>
<tr>
<th>Observations/Work Samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students felt confusion and experienced struggle to select and carry out a process.</td>
</tr>
<tr>
<td>Students took risks, making assumptions and trying solution strategies without being sure of their correctness.</td>
</tr>
<tr>
<td>Sometimes students sought approval from group members or the teacher as they attempted the strategies. Students admitted feeling somewhat uneasy.</td>
</tr>
<tr>
<td>Students established and verified correct solution paths, and felt more confident about their methods. Students experienced achievable challenge—the work was not beyond their capabilities, yet it still required them to think deeply and persevere, which they did.</td>
</tr>
<tr>
<td>Students came to realizations, understandings, and revelations that helped them justify their work and propel their work forward.</td>
</tr>
<tr>
<td>Revelations occurred as students pushed through struggle to explain and make sense of their work, often resolving the struggle.</td>
</tr>
<tr>
<td>Students' work was propelled forward when struggles were resolved and students came to important revelations. Students still described feelings of confusion, but began to describe feelings of confidence as well.</td>
</tr>
</tbody>
</table>

**Open Coding**

An element of confusion remained. Some students could form questions based on their experience with the task, but a few arrived again at an impasse.

Some students had correct answers on paper, but did not include a full explanation—missing a sketch, arithmetic, or a sentence.

**Axial Coding**

Students felt confusion and experienced struggle to select and carry out a process.

As students experimented with various solution strategies, they experienced struggle to carry out a process.

Middle of the Task: Students experienced struggle to carry out a process and struggle to make sense of their work.

Students' work was propelled forward when struggles were resolved and students came to important revelations.

Students still described feelings of confusion, but began to describe feelings of confidence as well.
Figure 4.3 Data Sorted in Levels of Coding for Research Question 1 – End of Task

<table>
<thead>
<tr>
<th>Coding</th>
<th>Raw Data</th>
<th>Open Coding</th>
<th>Axial Coding</th>
<th>Selective</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interviews</td>
<td>Observations ([Work Samples])</td>
<td>Students expressed feelings of pride, relief, happiness, and accomplishment as they came to final conclusions and made sense of the work.</td>
<td>Students experienced positive emotions and a positive self-concept after resolving the struggle to explain and make sense of their work.</td>
<td>Students confidently explained their solution paths, and expressed that the work became easier at the end of the task. Students identified strategies such as FOIL method and using diagrams which facilitated their success.</td>
</tr>
</tbody>
</table>

- "I felt relieved that I was done. I realized...that was really obvious what I was supposed to do." (A)
- "I felt pretty relieved that we finished it." (N)
- "Belief...I finished it and actually thought through it." (A)
- "I felt kind of accomplished." (N)
- "I'm pretty happy." (A)
- "I felt good about myself for just a small second." (A)
- "Positive vibes at the end." (A)

- "I most likely did everything right." (A)
- "Am I allowed to help other people?" [A]
- "Did figure it out? Oh my gosh." (A)

- "The end was the easiest for me." (N)
- "I realized...I was overthinking thing so much." (N)
- "It's easier if you break it down." (N)
- "Way easier." (A)
- "I used the diagrams and it made it easier." (A)
- "I'd say [part] C because once I got the two hard ones out of the way, C was very easy...cause I understood the concept more an it was just working out other correct areas." (A)
- "Not as easy are part A, but definitely easier than part B. So I guess it was middle of the road." (A)

- "I did x on one side and then the opposite end. So then I did another x on the opposite side, so I did four x's, which would be 4x. And then I went ahead and thought of something that would go with two of them that had to be the same to add to 12, so that was 12. And then the area for that I got was 2x^2 + 12x. (with confidence, sense of ease)"
- "Okay, 2 + times 2 = 4, and then we have to do the FOIL method (with confidence, sense of ease)."
- "So you would have to multiply 12 by 2x to get 24x, and that would make Alex's solution plausible (with confidence and ease)."
- "It's confirmed with Alex's 'cause the 12 and the 12s (confidence, ease)" (Mark thinks its 12^2 + 6^2 = 36, but Sarah is correct because Sarah has the 12x in the middle. After erase and revolving)

- "As a whole, I had completely understood the concepts. I'd completely conceptualized everything it was saying—what to do from [part] A to [part] C. I completely understood it.
- "It all really made sense at the end. From the beginning, it made no sense at all, but at the end you look back and it's like, okay, I kind of understand what to do now."
- "It seems a lot more clear because we're going over it with everyone." (N)
- "I definitely had to expand my thinking."
- "I understood it at the end."

- "It is an answer, I guess. On an Algebra II spectrum, I guess it does satisfy the requirements, because we did extract an answer for the sum of four and a product of five."
- "Okay, so we have plus or minus. So there are two answers." "Got it."
- "That was my own example to prove that was close. I solved that to see how close it was. And I noticed it was really close. I just needed to take out the x times the x." (A)
- "With a different possibility on perimeter a differing area is possible...You can change the numbers in different ways to get the same perimeter."
- "If I'm doing good because I know, since this is negative one, and it's squared, which equals positive one."
- "[With a differing possibility on side lengths, a differing area is plausible. It just has to add up to 12^2]"
- "[What can add to 4, multiply to 24]? (A)"
“It was really confusing, once we started discussing with our group and bouncing ideas off each other, it became a little bit more clear.”

“Once we grouped up it was easier to get going because we could talk with each other, about our ideas.”

“The end was easier for me because after we had all started talking and once we got all our ideas flowing in the group, then we could come together for an answer.”

“After we found the quadratic, finding and proving which was right was easier.”

“Once we started it out and we got that first piece, then it became easier.”

“Student number one helped us, guided us through it, and then I started to understand a little bit more.”

“After student one helped me through the solving part, I figured that bit was a little easier.”

“Student one had to help me out a little bit.”

“With my group work and my group help, I gained new thoughts that I wouldn’t have originally thought about or new tactics that I originally wouldn’t have thought about.”

“We all had a little bit of a different perspective on how to solve it, and when we combined it together, it made it—fell together, like a puzzle.”

“Working on our own helps us first, independent time before group time, because that allows each person to get their own ideas and I think that’s what makes the actual puzzle. We each get our own thoughts on it and then we get to share and then we get to put together each of our ideas, which can make a puzzle.”

“[Regarding going straight into group time if there was no independent time] We wouldn’t get as much done.”

“For Anna, how did you get that?” “Oh, I read the wrong thing. I didn’t mean to write that, it’s not—it’s not what she thinks it is a rectangle and not as a square.” “OK good.”

“S2: I guess technically two [solutions] because that’s negative one.” “T: I don’t think it’s because that’s negative one. Why is that two numbers?” “S2: OK so we have plus or minus, so there are two. “S2: Got it.” (Students correctly identify 2x as two solutions on their papers.)

“It’s negative one” “Negative one?” “Is it one or is it negative one?” “Negative one.” “Negative one, minus—so it would be positive.” (Students correctly rewrite subtracting f as adding b on their paper.)

“S12: If you factor it out, it’s x + 3 and x = 9.” “T: I’ll all see what she’s done? Could you put that on a rectangle”? “S10: Probable.” “S12: Yes.”

“So first we thought to combine all them, and then we were going to try and find what x was, then we realized we can’t really solve for x with what we already had. So we’re talking about how we could combine all this to get 24x.”

“Student one had to help me out a little bit.”

“Since that’s a positive five, you subtract five and set this equation equal to zero. Does that make sense?” “I’m trying to figure out currently how…?” “You have to see the equation equal to zero instead of five.” “Still five is positive, you subtract five and it cancels out on this side, but it becomes negative five on this side, because you have to balance the equation. Does that make sense?” “Yeah.”

“You’ll have to explain the same thing for me.” “So it’s plus and minus. So you have two separate answers.” “So it’s two?” “You have -2 and -2.” “Oh, okay.”

“Oh, so you said 2 and 12 on Alex, right? How did you get that?” “I know there couldn’t be x’s on both sides ‘cause that would be an x’ and that’s not what Alex got. So there could only be one x and then I also knew it had to be the half of 4 because it would have to add up to 4.” “Yeah, I got it.” “Okay.”

“Student 20 explained it as they would need s’s—I think I read that wrong. [corrects work on paper].”

“With my group work and my group help, I gained new thoughts that I wouldn’t have originally thought about or new tactics that I originally wouldn’t have thought about.”

“We all had a little bit of a different perspective on how to solve it, and when we combined it together, it made it—it fell together, like a puzzle.”

“Working on our own helps us first, independent time before group time, because that allows each person to get their own ideas and I think that’s what makes the actual puzzle. We each get our own thoughts on it and then we get to share and then we get to put together each of our ideas, which can make a puzzle.”

“[Regarding going straight into group time if there was no independent time] We wouldn’t get as much done.”

“odzi: That will multiply to 24 and none of it will add up to that.” “odzi: We can’t figure out anything that’s going to multiply to area but then also add together to the given answer (end of task)”

“You guys know what to do after this point? I don’t explicitly remember.” “I know there’s something.” “I know you have to divide—this seems wrong (end of task).”

As students discussed solution paths, misconceptions were resolved and newly formed understandings were clarified.

Students could test out each other’s strategies to confirm their validity.

The groups developed and tested strategies that were distinct from individual strategies.

The opportunity to discuss solutions with a small group made the task seem easier.

Certain students directly told group members their correct strategies.

Students felt this made the task easier.

Varying perspectives on the task enriched each students’ individual experience with the task.

Their progress on the task was impacted by shared perspectives.

Students valued the opportunity to form their own ideas before grouping up.

As students shared perspectives, they experienced and resolved struggle to explain and make sense of their work.

Varying perspectives on the task enriched each students’ individual experience with the task. Their progress on the task was impacted by shared perspectives. Students valued the opportunity to form their own ideas before grouping up.

Groups still reached impasses, especially as they started working on new parts of the task they had not yet encountered independently.

The high level of cognitive demand was not always removed by the opportunity to work with their group.
**Figure 4.5 Experiences of Advancing and Non-Advancing Students for Research Question 2 – Beginning of the Task**

<table>
<thead>
<tr>
<th>Student</th>
<th>Advancing or Non-Advancing</th>
<th>Struggle to Get Started</th>
<th>Struggle to Explain &amp; Make Sense of Work</th>
<th>Struggle to Carry out a Process</th>
<th>Struggle to Express Misconception/Error</th>
<th>Lost</th>
<th>Anxious</th>
<th>Persevered through confusion</th>
<th>Neutral/Positive</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>advancing</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>advancing</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>advancing</td>
<td></td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
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<td></td>
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<tr>
<td>9</td>
<td>advancing</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
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**Figure 4.6 Experiences of Advancing and Non-Advancing Students for Research Question 2 – Middle of the Task**

<table>
<thead>
<tr>
<th>Student</th>
<th>Advancing or Non-Advancing</th>
<th>Struggle to Carry out a Process</th>
<th>Resolved Struggle to Carry out a Process</th>
<th>Resolved Struggle to Explain &amp; Make Sense of Work</th>
<th>Confusion</th>
<th>Risk-taking</th>
<th>Achievable Challenge</th>
<th>Revelations</th>
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Patterns of Productive Struggle

Warshauer’s (2015a) four types of productive struggle were used as a framework to identify the patterns of productive struggle that students experienced. These struggle types were: struggle to get started, struggle to carry out a process, struggle to explain and make sense of their work, and struggle with a misconception or error (Warshauer, 2015a). In addition to these struggle types, the researcher sought to learn about the emotions students experienced throughout the phases of the task.

**Beginning of the task.** Students experienced all four types of struggle at the beginning phase of the task. Generally, students felt negative or neutral emotions during the beginning of the task, with only two students mentioning any slightly positive feelings.

Some students experienced a struggle to get started, expressing a sense of confusion, and reaching an impasse almost immediately. Some students had nothing written on their paper after several minutes, and when questioned by the researcher, said they did not know how to get
started. Students also mentioned feeling anxious, uneasy, frustrated, and overwhelmed. Some students experienced negative self-talk in this early phase of the task.

Some students experienced struggle to carry out a process and struggle to explain and make sense of their work. Some students admitted to feeling confused, but pressed on, attempting the work by utilizing various processes they developed or had previously learned. Students asked the researcher if their next step was correct, or even worked a problem correctly, then erased it when they were unsure of the validity of their method as they struggled to carry out a process. When the researcher asked them to explain their work, their questions surfaced, and students engaged in a struggle to explain and make sense of their work. Some students took a pragmatic approach, working without prompting from the researcher and without expressing negativity as they persevered through struggle to carry out a process as well as struggle to make sense of their work.

Some students misjudged the difficulty of the task or their level of understanding and did not feel confused immediately. Despite initial confidence, these students struggled with expressing a misconception or error. For example, several students did not think of a square as a special type of rectangle. One student subtracted when multiplication should have been used. Correcting these mistakes propelled their work forward.

**Middle of the task.** Students experienced struggle to carry out a process and struggle to make sense of their work during the middle of the task. These struggles also began to be resolved. Students still described feelings of confusion but began to describe more neutral feelings and feelings of confidence as well.

A struggle to carry out a process emerged as students expressed confusion about selecting a process to implement, forming questions based on their experience with the task. Some
students remained at an impasse, wrote answers on their paper without including a full explanation, or wrote question marks on their paper next to their work and responses.

Students experienced struggle to carry out a process when an element of risk-taking evolved as students made assumptions and tried solution strategies without being sure of their correctness. One student stated, “We had an idea, but we had no idea if that was actually the right direction.” Another student said, “I just went with it and hoped for the best.” Students admitted to feeling uneasy, and some sought approval from group members or the researcher before attempting their chosen strategies.

Student resolved the struggle to carry out a process as they established and verified correct solution paths and experienced and achievable challenge. Students said the problem started to make more sense. Their papers showed correct factorizations of the polynomials involved. Students began to express more confidence as their solution paths were verified.

Students experienced and resolved a struggle to explain and make sense of their work as they came to realizations and understandings that served to justify their solution strategies and propel their work forward. One student stated, “Then I realized two of the equations were quadratic, so I factored them to find length and width.” Another student corrected his justification when he realized one of the rectangles would not have an x on one side. “Actually, never mind, because this one, one of them won’t be x. I just know. One of the side lengths isn’t going to have an x in it.” These revelations occurred as students pushed through the struggle to explain and make sense of their work. Students exclaimed about their revelations: “Yes, yes, because area is length times width.” “Cool, I did math!” “Because that give you two—oh!” Students explained in the interviews that they began to understand the problem better, and it
seemed easier after a moment of revelation or insight, and they expressed more positive feelings and confidence.

**End of the task.** As students worked on the last parts of the task, they resolved their struggle to carry out a process and struggle to explain and make sense of their work. Students expressed positive emotions as the task concluded, and these struggles were resolved. One student summarized this feeling by stating, “Positive vibes at the end.”

Students expressed feelings of pride, relief, happiness, and accomplishment as they came to conclusions after they resolved the struggle to explain and make sense of their work. “I felt kind of accomplished,” said one student. Instead of negative self-talk, as at the beginning of the task, students mentioned feeling good about themselves and feeling smart. Clarity was evident in their work samples as well as verbal explanations. When the researcher asked one student how he was doing, the student responded, “Good, because I know, since this is negative one, and it’s squared, which equals positive one.” Also, students recognized their increased level of understanding as they resolved the struggle to explain and make sense of their work. One student explained, “At the end, you look back and it’s like, okay, I kind of understand what to do now.” Another student recognized that she had expanded her thinking. The struggle to explain and make sense was resolved, and students felt good about their accomplishments.

Students experienced confidence and a sense of ease as they resolved the struggle to carry out a process. Students confidently explained their solution paths. One student explained, “so you would have to multiply 12 by 2x to get 24x, and that would make Alex’s [solution] plausible,” their tone of voice indicating they felt sure of their explanation. Work samples showed incorrect conclusions erased and replaced by clearly stated correct conclusions. Students identified strategies such as drawing diagrams or using the FOIL method of polynomial
multiplication facilitated their success. Students stated that the work became easier at the end of
the task. “Once I got the two hard ones out of the way, C was very easy ‘cause I understood the
concept more and it was just working out other correct areas.” They resolved their process
struggle, and they identified their solution strategies with confidence.

**Working in groups.** Students referenced their time working in groups with enough
frequency during the interviews that it became apparent to the researcher that group work played
a significant role in the students’ experiences with productive struggle during the challenging
math task. Allowing time for group work both facilitated and resolved struggle to explain and
make sense of the work as well as struggle to carry out a process. There were instances when
group discussion might have reduced the level of cognitive demand of the task, but there were
other instances in which the group experienced struggle as a unit, and the level of cognitive
demand was maintained. The group discussion played a pivotal role in students’ developing a
deep understanding of the task’s solution paths as well as their feelings about their work on the
task.

The opportunity to discuss the task often made the task seem easier for students. One
student said, “Once we grouped up, it was easier to get going because we could talk to each other
about our ideas.” In small groups, students resolved misconceptions and clarified newly formed
understandings. “It was really confusing…once we started discussing with our group and
bouncing ideas off each other, it became a little bit more clear,” explained one student.

Once students grouped up, there was a greater potential for the level of cognitive demand
to be reduced, and this happened on several occasions when students helped each other by
directly telling their peers their correct strategies. For example, members of Student 1’s group
mentioned several times that Student 1 helped them understand the work. However, in other
instances, the entire group reached an impasse when they started working on new parts of the task they had not yet encountered independently. “We can’t figure out anything that’s going to multiply to area but then also add together to the given answer,” stated one student after the group had several minutes to work on the problem. Working with peers did not reduce the level of cognitive demand in all cases.

Discussing solution strategies with peers allowed students to both experience and resolve the struggle to explain and make sense of their work. Students could test out each other’s strategies to confirm their validity, and groups often developed and tested strategies that were distinct from their individual strategies. One student asked another, “For Anna, how did you get that?” testing the validity of their peer’s strategy. The peer then realized they had misread the question, and then explained the correct answer and how they came to it. The question from the original student engaged the peer in struggle to explain and make sense of their work. Another group tested a new strategy, distinct from any of their original strategies: finding a specific value for \(x\). Although they determined the strategy to be invalid, their discussion led them to another strategy, which turned out to be useful in solving the problem.

The group time, coupled with independent work time, allowed students to learn from their peers’ perspectives and develop enhanced strategies to solve the task. During the interview, students mentioned they valued the opportunity to work independently first. “Working on our own helps us first…because that allows each person to get their own ideas.” Students also recognized the value of hearing each other’s perspectives. One student stated, “With my group work…I gained new tactics that I originally wouldn’t have thought about.” Students mentioned that solution strategies and processes fell into place like pieces of a puzzle, and this was directly
observable throughout student-student interactions in the transcripts. The opportunity to share varying perspectives enriched each student’s experience with the task.

**Productive Struggle Patterns and Advancing Mastery Level**

Of the nine geometry students, five advanced to the next mastery level, and four did not. Two of the advancing students advanced from level 3 to level 4, and three of the advancing students advanced from level 2 to level 3. Two of the non-advancing students remained at level 3, one of the non-advancing students remained at level 2, and one of the non-advancing students dropped from level 2 to level 1. According to the data, advancing and non-advancing students had many of the same experiences throughout the math task, especially at the end. However, there were a few areas in which their experiences differed in the beginning and middle of the task.

**Beginning of the task.** Significant differences in the two groups at the beginning of the task included their experience with mistakes, struggle to express misconceptions or errors, and perseverance through confusion. Data showed evidence of three of the advancing students experiencing mistakes and struggle to express misconceptions or errors at the beginning of the task but showed just one of the non-advancing students experiencing mistakes. Data showed that four of the advancing students persevered through confusion at the beginning of the task but showed this for only one of the non-advancing students.

Slight differences between the two groups were evident in data for the struggle to explain and make sense of their work and struggle to carry out a process. All five advancing students experienced struggle to explain and make sense of their work, but just two non-advancing students experienced this at the beginning of the task. Regarding struggle to carry out a process,
four advancing students experienced this struggle, and two non-advancing students experienced this struggle.

Experiences with struggle to get started, feeling lost or anxious, or feeling neutral or even positive about the task occurred in the data with about the same frequencies between the groups at the beginning of the task.

**Middle of the task.** Differences between the two groups during the middle of the task centered around their experience with struggle to explain and make sense of their work, expressing confusion, having revelations, and risk-taking. The data showed more evidence of advancing students experiencing and resolving struggle to explain and make sense of their work than the non-advancing students. Data showed that all five advancing students experienced or resolved struggle to explain and make sense of their work, while only one non-advancing student had evidence of this in the data. Data showed four advancing students expressing confusion, while it only showed two non-advancing students expressing confusion. Data showed four advancing students experiencing a revelation that propelled their work forward, while only one non-advancing student had evidence of a propelling revelation. In the area of risk-taking, the data recorded all four non-advancing students taking risks within their solution paths, while only two of the advancing students took risks recorded in the data.

Experiences with struggle to carry out a process and achievable challenge occurred in the data with about the same frequencies between the groups throughout the middle of the task.

**End of the task.** The data showed little difference in the experiences of the advancing and non-advancing students at the end of the task. The data showed two students in each group resolving the struggle to explain and make sense of their work. For struggle to carry out a process, the data showed two advancing and three non-advancing students resolving the struggle.
The data showed two advancing students and one non-advancing student stating positive feelings at the end of the task. Three students in each group felt a sense of ease about the task at the end, according to the data. One student in each group expressed a sense of a more in-depth understanding of the task and its concepts at the end, according to the data.

**Summary of Research Findings**

The researcher examined the experiences of participants in this study through interviews, observations, and work samples. The design of the study allowed the researcher to examine the lived experience of bubble students as they work through a challenging math task. The coding process established types of productive struggle as well as emotions experienced by the participants.

For research question one, data showed unique characteristics of the beginning, middle, and end of the task. In the beginning, students experiencing all four types of struggle and had a negative emotional experience. In the middle of the task, students experienced the struggle to carry out a process and struggle to make sense of their work. Their emotional experience hit a turning point in the middle phase as revelations occurred which allowed them to proceed with their work. At the end of the task, students resolved their struggles and felt generally positive emotions. Group interactions played a significant role in the students’ experience during the middle and end phases of the task.

For research question two, disaggregation of data allowed comparison of bubble students who advanced to the next mastery level to those who did not advance. In the beginning phase of the task, advancing students tended to struggle with mistakes and misconceptions more than non-advancing students, but both groups experienced similarities in their experiences with struggle to get started and feeling lost or anxious. In the middle phase of the task, advancing students’
experience centered on a struggle to explain and make sense of their work and having revelation which propelled their work. Non-advancing students took more risks in the middle phase than advancing students as they experimented with various strategies to solve the problem. The end phase of the task did not reveal differences between the two groups.

The researcher analyzed the data and discussed the conclusions and implications in Chapter Five. A productive struggle progression and an emotional progression was developed and explained in Chapter 5. Additionally, differences between the experiences of advancing and non-advancing students were explored in more detail in Chapter 5.
CHAPTER FIVE: Conclusions, Implications, and Considerations

The purpose of this qualitative research study was to examine the experiences of students as they persisted with productive struggle on challenging math tasks. The design of the study consisted of classroom observations, semi-structured interviews, and student work samples. The researcher’s interest in the study stemmed from her observation of her colleagues’ desire to explore classroom practices that would improve student achievement, as well as the lack of research focused on the students’ perspective of productive struggle. The research conducted to answer the two research questions gave insight into bubble students’ overall experiences as well as delineated differences between the experiences of bubble students who advanced to the next mastery level and those who did not.

The conceptual framework centered on Warshauer’s (2015a) Productive Struggle Framework. This framework included four dimensions: Tasks, Student Struggle, Teacher Response, and Outcome (Warshauer, 2015a). This research study focused on the Student Struggle Dimension as well as the Outcome dimension. The Student Struggle dimension denoted the aspect of struggle the student experiences. These included getting started, carrying out a process, giving a mathematical explanation, and expressing a misconception or error (Warshauer, 2015a). In the Outcome dimension, the struggles were resolved either productively or unproductively (Warshauer, 2015). This study aimed to focus on productive struggle resolutions only.

Research Question 1

What patterns of productive struggle occur for bubble students as they work on challenging math tasks supporting mastery of instructional objectives in Algebra II and Geometry in one high school in Tennessee?
Findings for Research Question 1

Interviews, observations, and student work samples identified several patterns of productive struggle pertaining to the research question. Thirteen *bubble* students participated in this process. Four Algebra II students and nine Geometry students participated. The researcher obtained parent permission for all participants. The researcher used an open-coding process that led to the formation of axial codes, selective codes, and finally, themes within the data. The data revealed significant patterns regarding productive struggle types, the students’ emotional experience, and group work.

**Types of productive struggle.** Students experienced all four types of productive struggle throughout the challenging math tasks, with a shift in struggle types as students moved from one phase of the task to the next. Figure 5.1 illustrates the students’ struggle experience throughout the task.

*Figure 5.1 Student Struggle Progression Across Task Phases*

The student experience at the beginning of the task was most strongly characterized by a struggle to get started and a struggle with a misconception or error, but the struggle to carry out a
process and make sense of the work occurred as well. Struggle to get started and process struggle were typical experiences of students at the beginning of a task (Warshauer, 2015a).

In the middle and end of the task, data did not show students experiencing struggle to get started or struggle with misconceptions, but did show students continuing to experience struggle with processes and sense-making. Worth noting was many students’ struggle to accept that a square is a rectangle in this study. This struggle also occurred in Zeybek’s (2016) study. Some students began to resolve the process and sense-making struggles in the middle of the task, while others experienced struggle resolution in the end phase of the task.

Warshauer’s (2015a) research revealed the same struggles mid-task; however, her research identified misconception and error struggle at the end of the task rather than at the beginning. This discrepancy in struggle types may have occurred because this researcher may have prioritized clearing up misconceptions earlier in the task than other researchers. Struggle resolution upon task completion was determined to be productive because the data evidenced that students made statements and completed written work, which illustrated a deep understanding of the concepts involved in the task solution paths (Warshauer, 2015a).

**Emotional Experience.** A progression of emotional states was observed in the data as students moved from one phase of the task to another. Negative emotions largely characterized the student emotional experience at the beginning of the task, a feeling of acceptance of the challenge characterized the middle of the task, and positive emotions characterized the students’ experience as they neared completion of the task. Figure 5.2 illustrates this progression.
Students identified feelings of confusion and anxiety at the beginning of the task. A few felt positive and confident at first, but then entered a state of confusion as they started to progress into the task in the earliest stages. This initial confidence may have occurred because, at first glance, the numerical calculations seemed simple enough, but once students began work and realized what the task was asking, they recognized the true level of complexity. Students mentioned feeling lost and overwhelmed, characteristic of an initial struggle to get started (Warshauer, 2015a). A few students even referred to themselves as stupid, and a few did not write anything on their paper until the researcher asked them about their thoughts. Some students had ideas of what to do, but were hesitant to write them down for fear they may be wrong. Generally, the beginning of the task was a stressful time for students.

As the task progressed, these initial negative feelings began to dissipate. Some students remained confused and uneasy during the middle of the task, but in general, the negative emotions were not as intense, and they continued working. In the middle phase of the task, several students came to the point of revelation in which a realization occurred that allowed them to proceed more easily with the task, and their emotional state shifted. In a classroom where
productive struggle was the norm, students expected challenges and anticipated the need to persevere through frustration (Smith, as cited in NCTM, 2014). Students progressed at different rates in the middle of the task, but in general, a shift toward more neutral or positive emotions occurred.

At the end of the task, students felt positive emotions such as pride, a sense of accomplishment, and relief. Other researchers have observed that challenge and resulting success led to increased levels of dopamine in the brain, a neurotransmitter accompanied by a sense of pleasure and relaxation (Kienast et al., as cited in Willis, 2010), and the data suggested this occurred for participants in this study. Some students stated that they understood more about what the task was asking once they finished it, and expressed a sense of ease as they explained their answers. Now experts in the mathematical concepts involved in the task, students’ confidence at the end was a stark contrast to the hesitance they displayed at the beginning.

**Group Work.** The opportunity to work with a group was a significant part of the students’ experience with productive struggle in the task. The data revealed that the diverse perspectives of the other students in the group enriched students’ solution paths and understandings. According to Walshaw’s (2017) study, students working productively in groups were able to expand their thinking through interacting with peers by discussing, critiquing, and negotiating ideas and solutions, and the data shows evidence of this occurring for students in this research study.

Students stated that the opportunity to engage independently with the task before discussing it with their group supported the group work. Students developed their ideas independently, and then shared them with the group, so everyone in the group benefited from the individual’s independent think time. One student stated that if they had grouped up immediately,
“we wouldn’t get as much done [because of] people talking really and not focusing.” Another student described the process of processing each student’s ideas to create a final solution as being “like a puzzle,” in which each group member had unique pieces needed by the whole group to compose a complete understanding.

**Implications.** Teachers can prepare students to engage in productive struggle by letting them know what to expect from the process in terms of the struggle they will experience as well as the emotional progression they are likely to have. In previous research studies, when teachers expressed to students that struggle was expected, the classroom culture shifted to facilitate productive struggle and deep learning (Smith, as cited in NCTM, 2014). The data in this research study reveals a general progression of struggle types. If teachers explicitly teach students about the types of struggle they are likely to experience in each phase of the task, students may be more likely to persist through the struggle toward a productive resolution at the end of the task. The explicit teaching of struggle types and struggle progression is one aspect of a lesson that teachers can consider during lesson planning.

In addition to a struggle type progression, this research study revealed an emotional progression in the students’ experience. This progression began with negativity and anxiety, transitioned to acceptance of a challenge with lingering confusion, and finally shifted to relief and a sense of accomplishment. If teachers explicitly teach students about the emotional progression that is typical of a productive struggle experience, students may experience less anxiety and be able to approach the task more pragmatically. Knowledge of the emotional progression could be like knowledge of the stages of grief or stages of culture shock—when one knows what to expect during a time of grief, or while living in a new culture, the experience is less stressful. Teachers can instruct students to remain mindful of the typical emotional
progression as they work through the task, thereby making the mathematical challenges of the task more readily accessible as emotional stress does not cloud their thoughts.

The very existence of the struggle progression and emotional progression implies that teachers need to allow time for these progressions to fully develop if students are to receive the most significant benefit from challenging math tasks in the classroom. Teachers must pause the rush to cover content and patiently allow students to struggle through all phases of the task. Likewise, students need to be patient and realize that there will not be a quick solution. When both teachers and students have an awareness of the struggle progression and emotional progression, students will be more likely to engage deeply with the mathematical concepts of the task. It is not realistic for teachers to present students with a challenging math task daily, but when teachers use these tasks in the classroom, patience is critical.

The participants highlighted important implications for students working in groups on challenging math tasks. Students insightfully recognized the importance of independent work time before grouping up and noted that the various perspectives each student brought to the group helped the group come to a complete understanding of the task. Teachers may focus on the most actionable items in the current state evaluation rubric regarding grouping: the various grouping arrangements, roles and expectations for the group, and accountability. However, this study implies that preparation to work in a group is invaluable to the group’s success. In this study, the pre-work completed independently was vital to the productive outcome achieved by the group’s work. Instead of having students immediately group up for a problem-solving activity, this study implies that teachers should regularly implement independent work time before group time as a best practice. The practice of independent work time before grouping up may also be applicable in classes other than math.
Research Question 2

What is the relationship between patterns of productive struggle and mastery level shifts for bubble students in Geometry in one high school in Tennessee?

Findings for Research Question 2

Five geometry students advanced to the next mastery level, and four did not. Advancing and non-advancing students had some similar and some differing experiences at the beginning and middle phases of the task, while the two groups’ experiences at the end of the task were essentially the same. Figure 5.3 shows the general progression of the patterns of students’ experiences throughout the task.

Figure 5.3 Comparison of Experiences of Advancing and Non-Advancing Students

Types of struggle. The data gave evidence of more variation in struggle types in the group of advancing students as compared to the group of non-advancing students. The non-advancing students’ struggle experiences were generally limited to struggle to get started and carry out a process; in contrast, all four types of struggle showed up in the experiences of the advancing student group. Some advancing students resolved their struggle to explain and make
sense of the work in the middle of the task, while data did not evidence this struggle resolution for the non-advancing students until the end of the task. The data showed far less evidence of non-advancing students engaging in the struggle to explain and make sense of their work overall as compared to advancing students. The non-advancing students’ failures to resolve the struggle to get started and the process struggle could have prevented them from moving on to engage in other types of struggle on this task, and could be indicative of a pattern of non-engagement in struggle which impacted their eventual achievement mastery level.

**Characteristics of work.** Data showed differences in general characteristics of the work and solution pathways between advancing and non-advancing students. These differences centered on mistakes, misconceptions, risk-taking, and revelations.

The data showed advancing students struggling with mistakes and misconceptions early in the process, while there was little evidence of this happening with the non-advancing students. Sullivan and Mornane’s (2013) research noted that students who were not afraid to make mistakes in front of their peers or teacher were more likely to persist through a challenging task. Perhaps the advancing students were more expressive of these misconceptions, or more likely to ask for help when a mistake derailed their work, and this work habit could have impacted their eventual achievement mastery level.

Another variation between the two groups was the non-achieving students’ propensity for risk-taking as they sought solution paths. The advancing students were generally not documented taking mathematical risks, while the non-advancing students mentioned numerous times that they were trying strategies without confidence that the strategy was correct. While other research showed that risk-taking was generally to be encouraged throughout productive
struggle on a challenging math task (Livy, Muir, & Sullivan, 2018), it was notable that the data showed only the non-advancing students engaged in this practice on this task.

The data recorded advancing students experiencing realizations and revelations that propelled their work forward more often than non-advancing students. If this pattern occurred over time for the advancing students, the revelations could have impacted students’ eventual mastery level as they constructed their mathematical knowledge throughout the semester.

**Implications.** It is essential to recognize that this study did not focus on contrasts between high-achieving students and low-achieving students. Instead, this study focused on improvement. The advancing group included both high- and low-achieving participants; likewise, there were both high- and low-achieving participants that did not advance to the next mastery level. The study did not seek to find out what high-achievers do so that teachers can instruct all students to mimic their work habits. Instead, this study sought to learn about the productive struggle experiences of improving students—students who advance to the next mastery level at any level of achievement—to inform teachers of possible best practices to implement in their diverse classroom with the students’ perspective in mind.

An important implication of this study is the comfort level students had when revealing their mistakes or working through a misconception as it related to an improvement in achievement level. This study revealed a correlation between mistakes or misconceptions and advancement in achievement levels, and it was probably not the correlation one would expect. The students who were willing to admit and struggle through mistakes and misconceptions in the presence of their teacher and peers were the students who advanced to the next achievement level. Mistakes were associated with progress. The implication for teachers is to create a classroom culture where students are comfortable making mistakes and learning from them.
Another important characteristic of the advancing group was the variety in the types of struggle they displayed. Again, the result was counterintuitive—the group of students who experienced the greatest number of struggle types was the type that advanced in achievement level. Less varied struggle corresponded to less improvement in achievement level. Research showed that students’ willingness to persist in their learning (i.e., engage in productive struggle) impacted their level of achievement in a rigorous math course (Fung et al., 2018). One implication of this study is that willingness to engage in productive struggle impacts improvement in achievement level. Teachers can take action to encourage students to engage in productive struggle by offering challenging math tasks and explicitly informing students of the process of learning through productive struggle. Teachers could even teach students to identify their types of struggle, purposefully engaging in each struggle type throughout a challenging math task.

**Recommendations for Future Research**

This study shed light on how the researcher’s high school students experience productive struggle during challenging math tasks, but researchers could explore other ages or problem-solving situations. The study could be replicated with elementary students or middle school students to determine if they experience similar or different productive struggle progressions or emotional progressions when facing challenging math tasks. Similarly, further study could reveal whether high school students face similar or different struggle or emotional progressions with problem-solving challenges outside of math class.

Risk-taking is one area for possible future research. Teachers typically welcomed risk-taking in classrooms where productive struggle was the norm (Livy, Muir, & Sullivan, 2018), but in this case, the students who took risks when they explored solution paths did not advance to the
next mastery level. Perhaps the type of risks or the prior knowledge of students played a role in whether the students’ risk-taking was conducive to long-term learning. More research focused specifically on how students take mathematical risks when problem-solving could help explain the results of this study.

Finally, this study focused on thirteen students in one teacher’s classroom at one school. The study could be replicated to include a larger population of participants and various teachers at additional schools.

**Summary**

The findings of this study provided insight into the high school students’ experience with productive struggle. The researcher identified a progression of productive struggle, as well as a progression of emotional states. When the researcher compared *bubble* students who advanced to the next mastery level with those who did not, the data revealed some differences in their productive struggle patterns and emotional experience. These findings provide teachers with new ideas to support students in productive struggle and facilitate best practices, which lead to improved student achievement. It is the sincere hope of the researcher that these findings will be useful and applicable for classroom teachers as they seek to serve students each day in their classroom.
References


Appendix A

Parent Permission Form
Dear Parents and Guardians,

When students ask me why I wanted to become a teacher, I tell them it is because I love math and I love school. Two years ago, I decided to continue attending school by pursuing a doctoral degree. I am now an Ed. D. candidate at Carson-Newman University. As part of my degree completion requirements, I am conducting a research study of high school students to explore their experiences in “productive struggle” as they work on challenging math tasks. Sullivan County Schools has given me permission to conduct research in my classroom. This letter is to inform you about the study and ask your permission for your child to participate.

If you allow your child to participate, they may be asked to participate in several ways. First, audio recordings of their conversations with me (their teacher) and their classmates may be made as they work through challenging math tasks and discuss the task with me or their classmates. Second, some students will be asked to participate in small-group interviews regarding their experiences with the task. Audio recordings of these interviews will be made as well. Finally, student work samples may be selected to further support the research. Students frequently work on challenging math tasks in my classroom, so the activities they will be asked to do are already a part of our classroom routine. The only difference will be the audio recordings and interviews.

There are no known risks to your child from participating in this study. To maintain anonymity, pseudonyms will be used when transcribing audio recordings and examining student work samples. Students’ grades will not be affected in any way if they do or do not participate. Your child’s participation may benefit others by informing future classroom practices. I will be happy to answer any questions you have about the study. You can contact me at school at 423 354 1200 or by email at allison.marino@sullivank12.net.

Please sign and return the form below allowing your child to participate in the research study. I am fortunate to have a great group of students this semester as I pursue this endeavor. I greatly appreciate your help with my research and continued education.

Sincerely,

Allison Marino, Ed.S.

----------------------------------------------------------

Please check one box.

☐ I give permission for my child to participate in the study described above.

☐ I do not give permission for my child to participate in the study described above.

________________________________________
Print Child’s Name

________________________________________
Parent/Guardian’s Signature

________________________________________
Print Parent/Guardian’s Name

________________________
Date
Appendix B

Challenging Math Tasks
Rectangle Task

The perimeter of a rectangle is represented by $4x + 24$.

\[ P = 4x + 24 \]

In response to the question, “What is a possible representation of the area of this rectangle?” Mark says, “$x^2 + 36$,” and Sarah says, “No, $x^2 + 12x + 36$.”

A. What assumptions are both Mark and Sarah making about the rectangle? Comment on their answers.

Other students are also discussing the problem. Alex says that the area is $24x$. Anna says that she thinks the area is $x^2 + 12x + 27$.

B. Where do you think they are getting these answers? Can they all be correct?
Number Puzzle Task

A. Find two numbers whose sum is 4 and whose product is 5.

In part A, a quadratic equation was created.

B. Express the solutions to the quadratic equation you found in complex form.

C. Part A asked you to find two numbers whose sum is 4 and whose product is 5. Do these numbers satisfy these requirements? Why or why not?
Appendix C

Interview Questions
Interview Questions

A. I want to know what students are thinking and feeling at the beginning, middle, and end of the task.
   1. First, let’s talk about the beginning of the task. What is going through your mind as you get started?

   2. Now, let’s think back to the middle of the task, while you are working on it. What are you thinking and feeling?

   3. Finally, as you were finishing up the task and making final conclusions, what feelings are you experiencing? What are your thoughts at this point in the task?

B. I also want to know what was most challenging and what was least challenging about the task.
   a. What was the most challenging aspect of the task?
      i. What were you feeling as you worked through this challenge?

   b. What was the least challenging aspect of the task?
      i. What were you feeling as you worked through this portion of the task?